A survey on designs over finite fields

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ALCOMA15

dedicated to our friend Axel Kohnert (1962-2013)

recall results of the past 30 years

recall results of the past 30 years

focus on constructions

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focus on constructions

computer results group actions

recall results of the past 30 years

focus on constructions

provide list of known construtions

computer results group actions

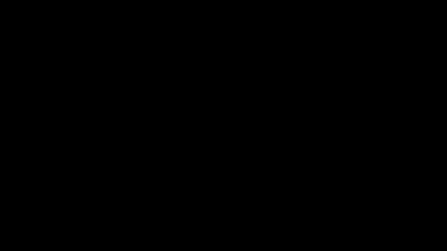
recall results of the past 30 years

focus on constructions

provide list of known construtions

computer results group actions

describe some new results



$$V$$
 set of cardinality n $P(V)$ lattice of subsets of V $\binom{V}{k}$ set of k -subsets of V

 $\operatorname{Aut}(P(V)) \simeq \operatorname{Sym}(V)$

 $Anti(P(V)) = \overline{\cdot} \circ Aut(P(V))$

$$\binom{n}{k}$$
 binomial coefficient

$$V$$
 set of cardinality n V n -dimensional vector space over \mathbb{F}_q $P(V)$ lattice of subsets of V $L(V)$ lattice of subspaces of V $\binom{V}{k}$ set of k -subsets of V $\binom{N}{k}$ binomial coefficient $\binom{n}{k}_q q$ -binomial coefficent $Aut(P(V)) \simeq Sym(V)$ $Aut(L(V)) \simeq P\Gamma L(V)$ $Anti(P(V)) = \overline{\cdot} \circ Aut(P(V))$ $Anti(L(V)) = \cdot^{\perp} \circ Aut(L(V))$

$$V$$
 set of cardinality n V n -dimensional vector space over \mathbb{F}_q $P(V)$ lattice of subsets of V $L(V)$ lattice of subspaces of V $\binom{V}{k}$ set of k -subsets of V $\binom{V}{k}$ set of k -subspaces of V $\binom{n}{k}$ binomial coefficient $\binom{n}{k}_q q$ -binomial coefficent $\operatorname{Aut}(P(V)) \simeq \operatorname{Sym}(V)$ $\operatorname{Aut}(L(V)) \simeq \operatorname{P}\Gamma L(V)$ $\operatorname{Anti}(P(V)) = \overline{\cdot} \circ \operatorname{Aut}(P(V))$ $\operatorname{Anti}(L(V)) = \cdot^{\perp} \circ \operatorname{Aut}(L(V))$

take any incidence structure on sets, replace sets by vector spaces, and cardinalities by dimensions



$$(V, \mathcal{B})$$
 is a t - (n, k, λ) design iff $V = \{1, \ldots, n\}, \mathcal{B} \subseteq \binom{V}{k}$ blocks $\forall T \in \binom{V}{t} : |\{B \in \mathcal{B} \mid T \subseteq B\}| = \lambda$

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 \dots and consider its q-analog

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number of blocks
$$|\mathcal{B}| = \lambda rac{igl[n]_q}{igl[k]_q}$$

trivial design
$$\mathcal{B} = {V \brack k}$$
 with $\lambda = {n-t \brack k-t}_q$

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Kiermaier, Pavčević, Intersection Numbers for Subspace Designs, 2014	B., <i>q</i> -Analogs of <i>t</i> -Balanced Designs of Finite Fields, 2014	over Laue, La	maier, Kohnert, arge Sets of e Designs, 2014		Association Schemes Igns in Regular es, 1976

Thomas, Designs

over Finite

Suzuki, 2-Designs

over $GF(2^m)$,

Ray-Chaudhuri, Schram, Designs

on Vectorspaces Constructed

B., Kerber, Laue, Systematic

Construction of q-Analogs of

Suzuki, 2-Designs

over GF(q),

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Thomas, Designs Geometries over 1996		Abe, Yoshiara, Or Construction of 2 over $GF(q)$, 1993	-Designs		ew Family of 2-Designs q) Admitting $SL_m(q^\ell)$,
Tits, Sur les Analogues Algébriques des Groupes Semi-Simples Complexes, 19	On a Class of	nemasa, Yoshiara, Small 2-Designs 995	Kiermaier, Laue and Residual Su Designs, 2014	bspace A	3., Kiermaier, Nakić, On the Automorphism Group of a Binary g-Analog of the Fano Plane, 2015
Fazeli, Lovett, Vardy, Non- trivial <i>t</i> -Designs over Finite Fields Exist for All <i>t</i> , 2013	B., Etzion, Östergå mann, On the Exis of Steiner Systems	tence of <i>q</i> -Analogs		of t-Designs	Etzion, Vardy, On <i>q</i> -Analogs of Steiner Systems and Covering Designs, 2011
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B., Kerber, Laue, Systematic
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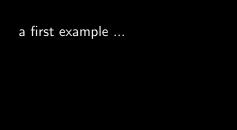
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a first example ... 1-(4,2,3;2)

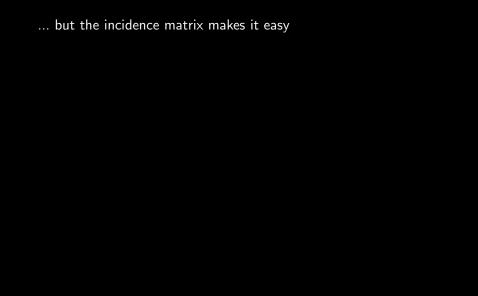
a first example \dots

1-(4, 2, 3; 2)

$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$	1 0	$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$	1 0	$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$	1 0	$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	0 1	$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$	0
	0	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	0	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	0 0		0 0	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	0 0
$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$	0 1 0	$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$	0 0 1	$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$	0 0 1	$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$	0 0 0 1
	0]	[0	0]	[1	0]	[1	0]	_1	
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	0	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	0	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
1	0	1	0	0	1	1	1	1	1

$\lceil 1 \rceil$	1]	$\lceil 1$	1]	$\lceil 1$	1	$\lceil 1$	0]	$\lceil 1$	0]
1	0	1	0	0	0	1	1	1	$1 \mid$
1	0	0	0	1	0	1	0	0	0
0	0]	[1	0]	[1	0]	0	0]	_1	0]
Γ0	0	Γ1	0	$\lceil 1$	[0	Γ0	0	[1	0
$\begin{vmatrix} 1 \\ 1 \end{vmatrix}$	1	1	0	0	0	1	0	1	0
	0	1	1	1	1	1	1	0	0
$\lfloor 1$	0]	0	0]	[1	0]	_1	0]	_1	1
Γ1	0	Γο	0	$\lceil 1$	17	Γ1	1	[0	1]
0	0	1	0	1	1	0	1	1	$1 \mid$
1	0	1	0	0	1	1	1	1	1
1	1	$\lfloor 1$	1	Lo	1	0	1	0	1

verification is quite uncomfortable ...



 \ldots but the incidence matrix makes it easy

0001,0111
0 0 1 1 0 1 1
0 0 1 1 1 0 1
0 0 1 0 0 1 1 1
0 0 1 0 1 0 1 1
0 0 1 0 1 1 1 0
0 1 0 0 0 0 1 1 1
0 1 0 1 1 0 1
0 1 0 0 1 1 1 0
1 0 0 1 0 1 1
1000
1000,1110
1 1 1 0 0 1 1
1 1 1 0 1 0 1
1 1 1 0 1 1 0

... but the incidence matrix makes it easy

1000 0100 0010 0001 1100 1010 1001 0110 0011 1110 1101 1011 0111 1111	
	0 0 0 1 0 1 1 1
	0 0 0 1 1 0 1 1
	0 0 1 1 1 0 1
	0 0 1 0 0 1 1 1
	0 0 1 0 1 0 1 1
	0 0 1 0 1 1 1 0
	0 1 0 0 0 0 1 1 1
	0 1 0 0 1 1 0 1
	0 1 0 0 1 1 1 0
	1000,1011
	1 0 0 0 1 1 0 1
	1 0 0 0 1 1 1 0
	1 1 1 0 0 1 1
	1 1 1 0 1 0 1
	1 1 1 1 0 1 1 0

... but the incidence matrix makes it easy

1 1 1 0 1 1 0	1 1	1
1 1 1 0 1 0 1	1	1
1 1 1 0 0 1 1	1	1
1 0 0 0 1 1 1 0	1	1
1 0 0 0 1 1 0 1	1 1	
1 0 0 0 1 0 1 1	1 1 1	
0 1 0 0 , 1 1 1 0	1	1
0 1 0 0 1 1 0 1	1 1 1	
0 1 0 0 0 1 1 1	1 1	
0 0 1 0 1 1 1 0	1	1
0 0 1 0 1 0 1 1	1 1 1	
0 0 1 0 0 1 1 1	1 1 1	
0 0 0 1 1 1 0 1	1 1	
0 0 1 1 0 1 1	1 1 1	
0001,01111	1 1	
	1000 0100 0010 0001 1100 1010 1001 0110 0011 1110 1101	0111 1111

... but the incidence matrix makes it easy

row sum is $\lambda = 3$ for all rows

now, group actions come into play \dots

extend action of $A = \operatorname{Aut}(L(V))$ to subsets of L(V) by $g\mathcal{B} := \{gB \mid B \in \mathcal{B}\}$

extend action of A = Aut(L(V))to subsets of L(V) by

isomorphism class of ${\cal B}$

$$g\mathcal{B} := \{gB \mid B \in \mathcal{B}\}$$

 $A(\mathcal{B}) := \{ g\mathcal{B} \mid g \in A \}$

extend action of A = Aut(L(V))to subsets of L(V) by $g\mathcal{B} := \{gB \mid B \in \mathcal{B}\}$

isomorphism class of ${\cal B}$

$$f \mathcal{B}$$

 $\hat{=}$ orbit

 $A(\mathcal{B}) := \{ g\mathcal{B} \mid g \in A \}$

automorphism group of
$$\mathcal{B}$$

 $\widehat{=}$ stabilizer
 $A_{\mathcal{B}} := \{ g \in A \mid g\mathcal{B} = \mathcal{B} \}$

to subsets of
$$L(V)$$
 by $g\mathcal{B} := \{gB \mid B \in \mathcal{B}\}$

isomorphism class of ${\cal B}$ $\hat{=}$ orbit

$$A(\mathcal{B}) := \{g\mathcal{B} \mid g \in A\}$$

Orbit-Stabilizer-Theorem

 $A(\mathcal{B}) \rightarrow A/A_{\mathcal{B}}$

$$A(\mathcal{B}) \rightarrowtail A/A_{\mathcal{B}}$$

 $A_{g\mathcal{B}} = gA_{\mathcal{B}}g^{-1}$

 $G = A_B = A_{gB} \Rightarrow g \in N_A(G)$

extend action of A = Aut(L(V))

automorphism group of \mathcal{B}

 $A_{\mathcal{B}} := \{ g \in A \mid g\mathcal{B} = \mathcal{B} \}$

isomorphism class of
$$\mathcal{B}$$
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Orbit-Stabilizer-Theorem
$$A(\mathcal{B}) \rightarrowtail A/A_{\mathcal{B}}$$

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$$G = A_{B} = A_{gB} \Rightarrow g \in N_{A}(G)$$

⇒ candidates for possible automorphism groups can be reduced to different conjugacy classes extend action of $A = \operatorname{Aut}(L(V))$ to subsets of L(V) by $g\mathcal{B} := \{g\mathcal{B} \mid \mathcal{B} \in \mathcal{B}\}$

automorphism group of \mathcal{B} $\widehat{=}$ stabilizer

 $A_{\mathcal{B}} := \{ g \in A \mid g\mathcal{B} = \mathcal{B} \}$

isomorphism class of \mathcal{B} $\widehat{=}$ orbit $A(\mathcal{B}) := \{g\mathcal{B} \mid g \in A\}$

Orbit-Stabilizer-Theorem

$$A(\mathcal{B}) \rightarrowtail A/A_{\mathcal{B}}$$

$$A_{g\mathcal{B}} = gA_{\mathcal{B}}g^{-1}$$

$$G = A_{\mathcal{B}} = A_{g\mathcal{B}} \Rightarrow g \in N_{\mathcal{A}}(G)$$

⇒ candidates for possible automorphism groups can be reduced to different conjugacy classes extend action of $A = \operatorname{\mathsf{Aut}}(L(V))$ to subsets of L(V) by $g\mathcal{B} := \{gB \mid B \in \mathcal{B}\}$

automorphism group of \mathcal{B} $\widehat{=}$ stabilizer $A_{\mathcal{B}} := \{g \in A \mid g\mathcal{B} = \mathcal{B}\}$

> ⇒ the normalizer of automorphims groups gives hints for classification issues (several papers by Laue)

our 1-(4, 2, 3; 2) design admits $G \simeq \text{Sym}(4)$ as group of automorphisms

our 1-(4, 2, 3; 2) design admits $G \simeq \text{Sym}(4)$ as group of automorphisms

	0001,0111	0001,1011	0001,1101	0010,0111	0010,1011	0010,1110	0100,0111	0100,1101	0100,1110	1000,1011	1000,1101	1000,1110	1111,0011	1111,0101	1111,0110
1000	1	1	1												
0100				1	1	1									
0010							1	1	1						
0001										1	1	1			
1100							1			1			1		
1010				1							1			1	
1001					1			1							1
0110	1	-							-			1		-	1
0101		1	1			1			1				1	1	
0011	1		1	1		1	1						1		
1110 1101 1011	1	1		1	1		1			1					
1011		1	1		_1			1		1	1				
0111						1		_	1		-	1			
1111									_			-	1	1	1

our 1-(4, 2, 3; 2) design admits $G \simeq \text{Sym}(4)$ as group of automorphisms

1000		0001,0111	0001,1011	0001,1101	0010,0111	0010,1011	0010,1110	0100,0111	0100,1101	0100,1110	1000,1011	1000,1101	1000,1110	1111,0011	1111,0101	1111,0110
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	0100 0010 0001 1100	1	1	1		1	1		1	1			1	1	1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1001 0110 0101	1	1	1	1	1	1		1	1		1	1	1		
	1110 1101 1011 0111	1	1		1	1		1	1	1	1	1	1			

our 1-(4, 2, 3; 2) design admits $G \simeq \text{Sym}(4)$ as group of automorphisms

	0001,0111	0001,1011	0001,1101	0010,0111 1	0010,1011	0010,1110	0100,0111 1	0100, 1101	0100,1110	1000,1011	1000,1101	1000,1110	1111,0011 1	1 1 1 1 0 1 0 1	1111,0110
1000 0100	1	1	1	_	_	_									
0100				1	1	1									
0010							1	1	1						
0001										1	1	1			
0001 1100 1010							1			1			1		
1010				1							1			1	
1001					1			1							1
0110	1											1			1
0110 0101		1							1					1	
0011			1			1							1		
1110	1			1			1								
1101		1			1					1					
1101 1011			1					1			1				
0111						1			1			1			
													-	-	-

our 1-(4,2,3;2) design admits $G \simeq \text{Sym}(4)$ as group of automorphisms

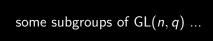
	000	0000	0000	0010	001	001	0100	010	010	1000	1000	100	1111	1 1 1	1 1 1
	0001,0111	0001,1011	0001,1101	0010,0111	0010,1011	0010,1110	0100,0111	0100,1101	0100,1110	1000,1011	1000,1101	1000,1110	1111,0011	1111,0101	11111,0110
1000	1	1	1			U			U			U	_		0
1000 0100	1	1	1	1	1	1									
0010				_	-	1	1	1	1						
0001										1	1	1			
1100 1010							1			$\dot{1}$			1		
1010				1							1			1	
1001					1			1							1
0110 0101	1											1			1
0101		1							1					1	
0011			1			1							1		
1110 1101	1			1			1								
1101		1			1					1					
1011			1					1			1				
0111						1			1			1			
1111													1	1	1



 $\begin{vmatrix} 2 & 1 \\ 3 & 0 \\ 0 & 3 \end{vmatrix}$

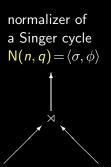
condensed *G*-incidence matrix
used to represent

design admitting G



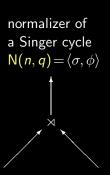
Singer cycle $S(n,q) = \langle \sigma \rangle \simeq \mathbb{F}_{q^n}^*$

Singer cycle Frobenius automorphism $S(n,q) = \langle \sigma \rangle \simeq \mathbb{F}_{a^n}^* \quad F(n,q) = \langle \phi \rangle \simeq \operatorname{Aut}(\mathbb{F}_{a^n}/\mathbb{F}_q)$



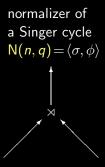
Singer cycle Frobenius automorphism $\mathsf{S}(n,q) = \langle \sigma \rangle \simeq \mathbb{F}_{q^n}^* \quad \mathsf{F}(n,q) = \langle \phi \rangle \simeq \mathsf{Aut}(\mathbb{F}_{q^n}\!/\mathbb{F}_q)$

if ℓ divides n then $\langle \sigma, \phi^{\ell} \rangle \simeq \mathsf{N}(n/\ell, q^{\ell})$ if n prime then $\mathsf{N}(n,q)$ is maximal in $\mathsf{GL}(n,q)$ $\mathsf{N}(n,q)$ is self-normalizing in $\mathsf{GL}(n,q)$



 $\begin{array}{ll} \text{Singer cycle} & \text{Frobenius automorphism} \\ \mathsf{S}(\textit{n},\textit{q}) = \langle \sigma \rangle \! \simeq \! \mathbb{F}_{q^n}^* & \mathsf{F}(\textit{n},\textit{q}) \! = \! \langle \phi \rangle \! \simeq \! \mathsf{Aut}(\mathbb{F}_{q^n}\!/\mathbb{F}_q) \end{array}$

if ℓ divides n then $\langle \sigma, \phi^{\ell} \rangle \simeq \mathsf{N}(n/\ell, q^{\ell})$ if n prime then N(n,q) is maximal in GL(n,q)N(n,q) is self-normalizing in GL(n,q)



Corollary

for primes n and q different t- $(n, k, \lambda; q)$ designs admitting N(n,q) as a group of automorphisms are non-isomorphic

Singer cycle
$$S(p,q) = \sqrt{q} \sim \mathbb{R}^*$$

Frobenius automorphism $\mathsf{S}(n,q) = \langle \sigma \rangle \simeq \mathbb{F}_{a^n}^* \quad \mathsf{F}(n,q) = \langle \phi \rangle \simeq \mathsf{Aut}(\mathbb{F}_{a^n}/\mathbb{F}_a)$

$$S_r(T) := \langle b_0 \dots b_{r-1} \mid b_i \in T, 0 \le i < r \rangle$$

$$\mathcal{B}_r := \{ S_r(T) \mid T \in {V \choose 2} \}$$

$$S_r(T) := \langle b_0 \dots b_{r-1} \mid b_i \in T, 0 \le i < r \rangle$$

 $\mathcal{B}_r := \{S_r(T) \mid T \in \begin{bmatrix} v \\ 2 \end{bmatrix} \}$

Theorem (Suzuki, 1990)

for $n \geq 7$ with (n, 4) = 1 the set \mathcal{B}_2 defines a 2- $(n, 3, \begin{bmatrix} 3 \\ 2 \end{bmatrix}_a; q)$ design admitting N(n,q) as a group of automorphisms

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under which conditions does \mathcal{B}_r define a 2- $(n, r+1, {r+1 \brack 2}_q; q)$ design?

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Theorem (Abe, Yoshiara, 1993) the set
$$\mathcal{B}_r$$
 is no design for $q=2$ and $4 \le r+1 < n \le 15$ except for the pair $r=3$, $n=7$

$$S_r(T) := \langle b_0 \dots b_{r-1} \mid b_i \in T, 0 \le i < r \rangle$$

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Theorem (Suzuki, 1990)

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Theorem (Abe, Yoshiara, 1993) the set \mathcal{B}_r is no design for q=2 and $4 \le r+1 < n \le 15$ except for the pair r=3, n=7

conjecture: \mathcal{B}_3 is the dual of \mathcal{B}_2 for n=7 and all q

Theorem (Kramer, Mesner, 1976) there exists a t- $(n, k, \lambda; q)$ design

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iff \exists 0-1 solution $x = [\dots, x_K, \dots]^t$ of

Theorem (Kramer, Mesner, 1976) there exists a t- $(n, k, \lambda; q)$ design

there exists a
$$t$$
- (n, κ, λ, q) design

iff
$$\exists$$
 0-1 solution $x = [\dots, x_K, \dots]^t$ of

with

 $\zeta_{t,k} = (\zeta_{TK})$

Theorem (Kramer, Mesner, 1976) there exists a t- $(n, k, \lambda; q)$ design

iff
$$\exists$$
 0-1 solution $x = [\dots, x_K, \dots]^t$ of

$$\begin{bmatrix} V \\ t \end{bmatrix} \ni \qquad T \qquad \begin{bmatrix} K & \in & \begin{bmatrix} V \\ k \end{bmatrix} \\ \vdots \\ K & \in & \begin{bmatrix} V \\ k \end{bmatrix} \end{bmatrix} \vdots \\ X \downarrow K & \in & \begin{bmatrix} V \\ k \end{bmatrix} \\ \vdots \\ \lambda & \vdots \\ \lambda &$$

with

 $\overline{\zeta_{t,k}} = (\zeta_{TK})$

Theorem (Kramer, Mesner, 1976) there exists a t- $(n, k, \lambda; q)$ design admitting $G \leq \operatorname{Aut}(L(V))$ as a group of automorphisms iff

$$\exists$$
 0-1 solution $x = [\dots, x_K, \dots]^t$ of

with

 $\zeta_{t,k}^{\mathsf{G}} = (\zeta_{\mathsf{T}K}^{\mathsf{G}})$

$$G(K) \in G \setminus \begin{bmatrix} V \\ k \end{bmatrix} \\ \vdots \\ C \setminus \begin{bmatrix} V \\ t \end{bmatrix} \ni G(T) \quad \vdots \\ \cdots \quad \zeta_{TK}^{G} \quad \vdots \\ \vdots \\ \lambda \quad \vdots$$

$$\zeta_{TK}^G = \sum_{K' \in G(K)} \zeta(T, K')$$



2-(7, 3, 7; 2)

2-(7,3,7;2)

plain incidence matrix $\zeta_{2,3}$ 2667 × 11811

2-(7,3,7;2)

plain incidence matrix $\zeta_{2,3}$ 2667 \times 11811

prescribed group of automorphisms normalizer of a singer cycle N(7, 2) of order 889

2-(7,3,7;2)

plain incidence matrix $\zeta_{2,3}$ 2667 \times 11811

prescribed group of automorphisms normalizer of a singer cycle N(7, 2)of order 889

we consider Suzuki's design...

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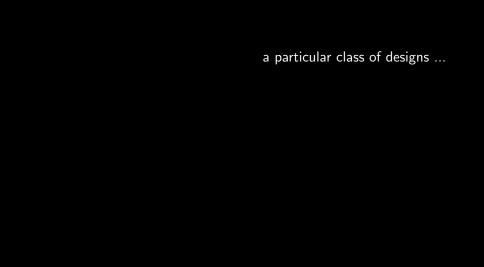
we consider Suzuki's design...

2-(7,3,7;2)

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prescribed group of automorphisms normalizer of a singer cycle N(7, 2)of order 889

19 solutions (non-isomorphic)



$$\begin{array}{c} \text{t-}(\textit{n},\textit{k},\lambda;\textit{q}) \text{ design } \mathcal{B} \text{ transitive} \\ \text{iff } |A_{\mathcal{B}} \backslash \! \backslash {V \brack 1}| = 1 \end{array}$$

$$egin{aligned} t ext{-}(n,k,\lambda;q) ext{ design } \mathcal{B} ext{ transitive } \ & ext{iff } |A_{\mathcal{B}}ackslashig[{Vtop 1}]|=1 \end{aligned}$$

Theorem (Miyakawa, Munemasa, Yoshiara, 1995) \mathcal{B} non-trivial transitive t- $(n, k, \lambda; q)$ design, $t \geq 2$ - if n = 6 then $q \equiv 1 \mod 3$ and $S(6, q) \not\leq A_{\mathcal{B}} \leq N(6, q)$

- if n prime then either $A_{\mathcal{B}} = S(n,q)$ or $A_{\mathcal{B}} = N(n,q)$

$$t ext{-}(n,k,\lambda;q)$$
 design $\mathcal B$ transitive iff $|A_{\mathcal B}ackslashigl[{Vtop 1}]|=1$

Theorem (Miyakawa, Munemasa, Yoshiara, 1995) \mathcal{B} non-trivial transitive t- $(n, k, \lambda; q)$ design, $t \geq 2$ – if n = 6 then $q \equiv 1 \mod 3$ and $S(6, q) \not\leq A_{\mathcal{B}} \leq N(6, q)$ – if n prime then either $A_{\mathcal{B}} = S(n, q)$ or $A_{\mathcal{B}} = N(n, q)$

⇒ Suzuki's design is transitive

$$\text{t-}(n,k,\lambda;q) \text{ design } \mathcal{B} \text{ transitive } \\ \text{iff } |A_{\mathcal{B}} \backslash \! \backslash {V \brack 1}| = 1$$

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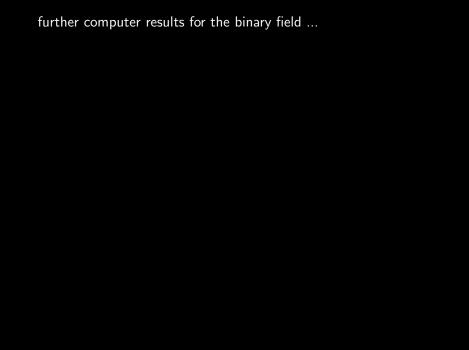
⇒ Suzuki's design is transitive

Theorem (Miyakawa, Munemasa, Yoshiara, 1995)

$$[\lambda, \text{number of non-isom. } 2-(7,3,\lambda;q) \text{ designs } \mathcal{B} \text{ with } A_{\mathcal{B}} = \mathbb{N}(n,q)]$$

$$q=2$$
: [3,2], [5,14], [7,19], [10,30], [12,90], [14,55], [λ ,0] for $\lambda=1,2,4,6,8,9,11,13,15$

$$q=$$
 3: [5,22], [λ ,0] for $\lambda=$ 1,2,3,4



```
further computer results for the binary field ...
```

Theorem (B., Kerber, Laue, S. Braun, 2005–2015) $|\zeta_{t,k}^G|$ t- $(n, k, \lambda; q)$ $3-(8, 4, \lambda; 2)$ N(8, 2) 53×109 11 $N(4, 2^2)$ 105×217 11. 15 2-(11, 3, λ ; 2) N(11, 2) 31×2263 245, 252 2-(10, 3, λ ; 2) N(10, 2) 20×633 15, 30, 45, 60, 75, 90, 105, 120 $2-(9,4,\lambda;2)$ N(9,2) 11×725

966, 1008, 1029, 1071, 1092, 1134, 1155,

336, 378, 399, 441, 462, 504, 525, 567, 588, 630, 651, 693, 714, 756, 777, 819, 840, 882, 903, 945, 1197, 1218, 1260, 1281, 1323 N(9, 2) 11×177 63 $N(3, 2^3)$ 31×529 21, 22, 42, 43, 63

 $N(8,2) \times 1$ 28×408 7, 12, 19, 24, 31, 36, 43, 48, 55, 60 $M(3, 2^3)$ 40×460 49 15×217

 $2-(8, 4, \lambda; 2)$ $N(4, 2^2)$ 21, 35, 56, 70, 91, 105, 126, 140, 161, 175, 196, 210, 231, 245, 266, 280, 301, 315 $N(7,2) \times 1 \quad 13 \times 231$ 7, 14, 49, 56, 63, <u>98,</u> 105, <u>112,</u> 147, <u>154,</u> 161,

3, 5, 7, 10, 12, 14,

196, 203, 210, 245, 252, 259, 294, 301, 308

3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

 $2-(9,3,\lambda;2)$

21

3, 6

 15×105

 3×15

 21×93

 77×155

 $2-(8,3,\lambda;2)$

 $2-(7,3,\lambda;2)$

 $2-(6,3,\lambda;2)$

 $N(4, 2^2)$

N(7, 2)

S(7, 2)

 $\langle \sigma^7 \rangle$

21, 63, 84, 126, 147, 189, 210, 252, 273, 315,

t- $(n, k, \lambda; q)$ G

... also for q=3, 4 and 5

2-(8, 3, λ ; 3)	N(8, 3)
2- $(7, 3, \lambda; 3)$	N(7, 3)

 $2-(6,3,\lambda;3)$

 $2-(6,3,\lambda;4)$

 $2-(6,3,\lambda;5)$

$$(7,3)$$
 13×121 $(7,4)$ $(7,3)$ 25×76

$$\langle \sigma^2, \phi^2 \rangle$$

 $\langle \sigma^{17}, \phi \rangle$

 $\langle \sigma^3, \phi \rangle$

N(6,5)

 $\langle \sigma^3, \phi \rangle \times 1$

$$\langle \sigma^{17}, \phi \rangle$$
 93 × 234
S(5,3)×1 51 × 150

$$\langle \sigma^1, \phi \rangle$$
 93
S(5,3)×1 51
 $\langle \sigma^2 \rangle \times 1$ 91

 $|\zeta_{t,k}^G|$

 41×977

$$\frac{91 \times 280}{51 \times 161}$$

 53×248

78

λ

20

16

8, 16, 20

52, 104, 156

$$t$$
- $(n, k, \lambda; q)$

$$t-(n,k,\begin{bmatrix}n-t\\k-t\end{bmatrix}_q-\lambda;q)$$
 supplemented
$$t-(n,k,\lambda;q)$$

$$t\text{-}(n,k,\left[\begin{smallmatrix} n-t \\ k-t \end{smallmatrix} \right]_q - \lambda;q)$$
 supplemented
$$t\text{-}(n,k,\lambda;q)$$
 derived
$$(t-1)\text{-}(n-1,k-1,\lambda;q)$$

 $t-(n, k, {n-t \brack k-t}_q - \lambda; q)$

supplemented

(V, \mathcal{B}) t- $(n, k, \lambda; q)$ design

t- $(n, k, \lambda; q)$

$$(V, B)$$
 t- $(n, k, \lambda; q)$ design $i + i < t$. $P \in [V]$. $H \in I$

derived

$$i + j \le t$$
, $P \in \begin{bmatrix} V \\ i \end{bmatrix}$, $H \in \begin{bmatrix} V \\ n-j \end{bmatrix}$

$$(V,B)$$
 $t-(N,K,X,Q)$
 $i+j \leq t, P \in \begin{bmatrix} V \\ i \end{bmatrix}$

$$i + j \le t, P \in {V \brack i}, H \in {V \brack n-j}$$

$$\Rightarrow |\{B \in \mathcal{B} \mid P \le B \le H\}| = \lambda \frac{{n-j-i \brack k-i}_q}{{n-1 \brack k-i}_q}$$

$$\{ , \lambda ; q \}$$
 design $\{ ar{V} \}$. $H \in \mathbb{R}$

(t-1)- $(n-1, k-1, \lambda; q)$

design
$$H \in [$$



$$\lambda; q)$$
 desi

$$\lambda; q)$$
 designation

$$\lambda; q)$$
 des

$$q$$
) des

$$\lambda; q)$$
 des

$$(\lambda,\lambda;q)$$
 de

$$,\lambda;q)$$
 de

$$(a,\lambda;q)$$
 de

Theorem (Suzuki, 1989)
$$(V, B)$$
 t - $(n, k, \lambda; a)$ design

(Suzuki,
$$(n, k, \lambda; a)$$

 $t-(n, k, {n-t \brack k-t}_q - \lambda; q)$

(i=t-1, j=0) reduced

(t-1)- $(n, k, \lambda \frac{q^{n-t+1}-1}{a^{k-t+1}-1}; q)$

t- $(n, k, \lambda; q)$

$$\Rightarrow |\{B \in \mathcal{B} \mid P \leq B \leq H\}| = \lambda \frac{{\binom{n-t}{k-t}}_q^{-t}}{{\binom{n-t}{k-t}}_q}$$

(t-1)- $(n-1, k-1, \lambda; q)$

$$D \geq H$$

$$\leq H$$

$$\leq H$$

$$\leq H$$

$$3 \leq F$$

$$\leq H$$

$$B \leq F$$

$$8 \leq F$$

$$3 \leq F$$

 $i + j \le t, P \in {V \brack i}, H \in {V \brack n-j}$

Theorem (Suzuki, 1989) (V, \mathcal{B}) t- $(n, k, \lambda; q)$ design

derived

further constructions Theorem (Suzuki, 1989) (V, \mathcal{B}) t- $(n, k, \lambda; q)$ design $i+j \leq t, P \in {V \brack i}, H \in {V \brack n-i}$ $t-(n,k,\begin{bmatrix}n-t\\k-t\end{bmatrix}_q-\lambda;q)$ $\Rightarrow |\{B \in \mathcal{B} \mid P \leq B \leq H\}| = \lambda \frac{\lfloor \frac{n-1}{2} \rfloor_q}{\lfloor \frac{n-1}{2} \rfloor}$ supplemented t- $(n, k, \lambda; q)$ (i=t-1, j=0) reduced derived

(t-1)- $(n-1, k-1, \lambda; q)$

$$(i=0, j=t, \cdot^{\text{dual}})$$

$$t-(n, n-k, \lambda {n-t \brack k}_q/{n-t \brack k-t}_q; q)$$

(t-1)- $(n, k, \lambda \frac{q^{n-t+1}-1}{q^{k-t+1}-1}; q)$

further constructions Theorem (Suzuki, 1989) (V, \mathcal{B}) t- $(n, k, \lambda; q)$ design $i+j \leq t, P \in {V \brack i}, H \in {V \brack n-i}$ $t-(n, k, {n-t \brack k-t}_q - \lambda; q)$ $\Rightarrow |\{B \in \mathcal{B} \mid P \leq B \leq H\}| = \lambda \frac{{\binom{n-1}{k-1}}_q}{{\binom{n-1}{k-1}}_q}$ supplemented t- $(n, k, \lambda; q)$ derived (i=t-1, j=0) reduced (t-1)- $(n, k, \lambda \frac{q^{n-t+1}-1}{q^{k-t+1}-1}; q)$ (t-1)- $(n-1, k-1, \lambda; q)$

$$(i=0,j=t,\cdot^{\perp}) / (i=t-1,j=1)$$

$$t-(n,n-k,\lambda{n-t \brack k}_q/{n-t \brack k-t}_q;q)$$

$$(t-1)-(n-1,k,\lambda\frac{q^{n-k}-1}{q^{k-t+1}-1};q)$$

residual

Theorem (Kiermaier, Laue, 2014) the existence of designs for derived and residual parameters implies

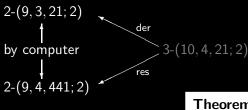
the existence of a design for the reduced parameters

```
2-(9,3,21;2)

thickness by computer
2-(9,4,441;2)
```

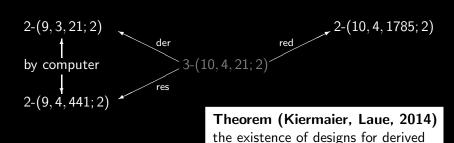
Theorem (Kiermaier, Laue, 2014) the existence of designs for derived and residual parameters implies the existence of a design

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for the reduced parameters

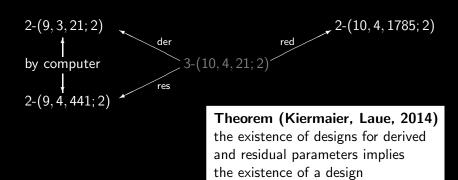


and residual parameters implies

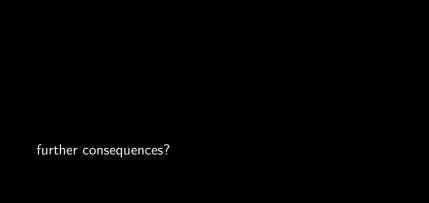
the existence of a design for the reduced parameters

Corollary (Kiermaier, Laue, 2014)

There exist designs with parameters 2-(8, 4, λ ; 2) for $\lambda = 63$, 84, 147, 168, 189, 252, 273, 294 2-(10, 4, λ ; 2) for $\lambda = 1785$, 1870, 3570, 3655, 5355 2-(8, 4, 91 λ ; 3) for $\lambda = 5$, 6, 7, . . . , 60



for the reduced parameters

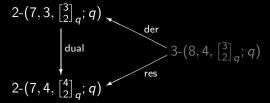


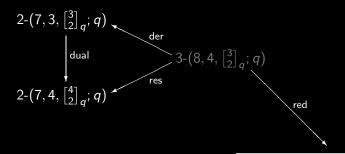
$$2-(7,3, [\frac{3}{2}]_q; q)$$

$$2-(7,3,\begin{bmatrix}3\\2\end{bmatrix}_q;q)$$

$$\downarrow^{\text{dual}}$$

$$2-(7,4,\begin{bmatrix}4\\2\end{bmatrix}_q;q)$$

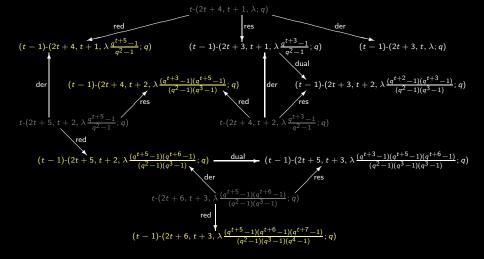




further consequences?

Corollary there exists a family of $2-(8,4,\frac{(q^6-1)(q^3-1)}{(q^2-1)(q-1)};q)$ designs for all prime powers q

$$(t-1)$$
- $(2t+3, t+1, \lambda \frac{q^{t+3}-1}{q^2-1}; q)$ $(t-1)$ - $(2t+3, t, \lambda; q)$



$$(t-1) - (2t+4,t+1,\lambda;q)$$

$$(t-1) - (2t+4,t+2,\lambda;q)$$

$$(t-1) - (2t+4,t+2,\lambda;q)$$

$$(t-1) - (2t+4,t+2,\lambda;q)$$

$$(t-1) - (2t+4,t+2,\lambda;q)$$

$$(t-1) - (2t+3,t+2,\lambda;q)$$

$$(t-1) - (2t+4,t+2,\lambda;q)$$

$$(t-1) - (2t+5,t+3,\lambda;q)$$

$$(t-1) - (2t+6,t+3,\lambda;q)$$

$$(t-1) - (2t+6,t+3,\lambda$$

Corollary

There exist designs with parameters 2-(10, 4, 85 λ ; 2), 2-(10, 5, 765 λ ; 2), 2-(11, 5, 6205 λ ; 2), 2-(12, 6, 423181 λ ; 2) for $\lambda = 7, 12, 19, 21, 22, 24, 31, 36, 42, 43, 48, 49, 55, 60, 63$

computer results for t = 3 and q = 2

Theorem (Itoh, 1998)

there is a family of 2- $(n\ell,3,q^3\frac{q^{n-5}-1}{q-1};q)$ designs admitting $\mathrm{SL}(\ell,q^n)$ as group of automorphisms for $q\geq 2,\ \ell\geq 3,\ n\equiv 5\ \mathrm{mod}\ 6(q-1)$

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there is a family of 2- $(n\ell,3,q^3\frac{q^{n-5}-1}{q-1};q)$ designs admitting $\mathrm{SL}(\ell,q^n)$ as group of automorphisms for $q\geq 2$, $\ell\geq 3$, $n\equiv 5 \mod 6(q-1)$

$$\zeta_{2,3}^{\mathsf{SL}(\ell,q^n)} = \left[egin{array}{c|cccc} \zeta_{2,3}^{\mathsf{S}(n,q)} & a & & & & & & \\ \hline \zeta_{2,3}^{\mathsf{SL}(\ell,q^n)} & \ddots & & 0 & & 0 \\ \hline & 0 & X & Y & Z \end{array} \right]$$

 $\overline{2-(n,3,\lambda;q)}$ designs admitting S(n,q) can be extended to $2-(n\ell,3,\lambda;q)$ designs admitting $SL(\ell,q^n)$ if λ is appropriate

Theorem (Itoh, 1998)

there is a family of 2- $(n\ell, 3, q^3 \frac{q^{n-5}-1}{q-1}; q)$ designs admitting $SL(\ell, q^n)$ as group of automorphisms for $q \geq 2$, $\ell \geq 3$, $n \equiv 5 \mod 6(q-1)$

extension possible if

$$\lambda = q(q+1)(q^3-1)s + q(q^2-1)t$$

for some integer $s \ge 0$, $t \in \{0,1\}$ if 3|n and t = 0 otherwise

extension possible if

$$\lambda = q(q+1)(q^3-1)s + q(q^2-1)t$$

for some integer $s \ge 0$, $t \in \{0,1\}$ if 3|n and t = 0 otherwise

for
$$\ell \geq 3$$
, $n \equiv 5 \mod 6(q-1)$
Suzuki's supplemented design
has index value λ of required form

further results from Itoh's construction?

$$q = 2$$
: $\lambda = 42s + 6t$
 $q = 3$: $\lambda = 312s + 24t$

further results from Itoh's construction?

```
computer constructions
of designs admitting S(n, q)
```

further results from Itoh's construction?

2-(8, 3, 21; 2)
$$\stackrel{\text{supp}}{\Rightarrow}$$
 2-(8, 3, 42; 2)
2-(9, 3, 42; 2)
2-(9, 3, 43; 2) $\stackrel{\text{supp}}{\Rightarrow}$ 2-(9, 3, 84; 2)

 $2-(10, 3, 45; 2) \stackrel{\text{supp}}{\Rightarrow} 2-(10, 3, 210; 2)$ 2-(13, 3, 42s; 2) for 1 < s < 25

$$(3, 45; 2) \stackrel{\text{supp}}{\Rightarrow} 2 - (10, 3, 210; 3, 42s; 2) \text{ for } 1 \le s \le 25$$

 $2-(8,3,52;3) \stackrel{\text{supp}}{\Rightarrow} 2-(8,3,312;3)$

$$f^2 = (3, 3, 3, 3, 2)$$

or $1 < s < 25$

a = 3: $\lambda = 312s + 24t$





q = 2: $\lambda = 42s + 6t$

computer constructions of designs admitting S(n, q)

further results from

Itoh's construction?

2-(9, 3, 42; 2) $2-(9,3,43;2) \stackrel{\text{supp}}{\Rightarrow} 2-(9,3,84;2)$

2-(13, 3, 42s; 2) for 1 < s < 25 $2-(8,3,52;3) \stackrel{\text{supp}}{\Rightarrow} 2-(8,3,312;3)$

 $2-(10, 3, 45; 2) \stackrel{\text{supp}}{\Rightarrow} 2-(10, 3, 210; 2)$

 $2-(8,3,21;2) \stackrel{\text{supp}}{\Rightarrow} 2-(8,3,42;2)$

a = 3: $\lambda = 312s + 24t$

q = 2: $\lambda = 42s + 6t$

Corollary there exist families of designs

admitting $SL(\ell, q^n)$ for $\ell > 3$

 $2-(8\ell, 3, 42; 2)$ $2-(9\ell, 3, 42s; 2)$ for 1 < s < 2 $2-(10\ell, 3, 210; 2)$ $2-(13\ell, 3, 42s; 2)$ for 1 < s < 25 $2-(8\ell, 3, 312; 3)$

$$S_q(t, k, n)$$
 q-Steiner system
= t - $(n, k, 1; q)$ design

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Theorem

 $S_q(1, k, n)$ exists if and only if k divides n

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Theorem

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trivial q-Steiner systems

$$S_q(t, k, n)$$
 q-Steiner system
= t - $(n, k, 1; q)$ design

Theorem

 $S_q(1, k, n)$ exists if and only if k divides n

trivial q-Steiner systems

existence of non-trivial $q ext{-Steiner}$ systems (t>1) was long-standing issue ...

Theorem (B., Etzion, Östergård, Vardy, Wassermann, 2012) *q*-Steiner systems do exist

Theorem (B., Etzion, Östergård, Vardy, Wassermann, 2012) q-Steiner systems do exist

 $S_2(2,3,13)$

Theorem (B., Etzion, Östergård, Vardy, Wassermann, 2012) *q*-Steiner systems do exist

$$S_2(2,3,13)$$

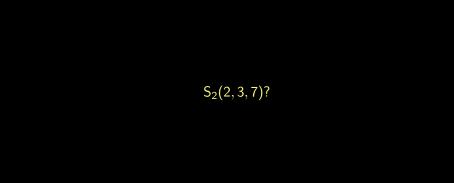
automorphism group N(13, 2) of order $13 \cdot (2^{13} - 1) = 106483$

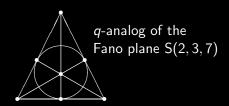
consisting of 15 orbits of full length 106483

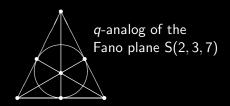
taken out of 25572 possible orbits

≥ 1050 disjoint solutions all non-isomorphic

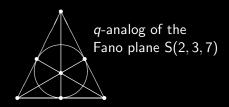
solved with Knuth's dancing links





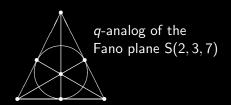


it would have size 381



it would have size 381

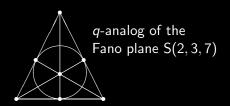
replacing "exactly" by "at most" yields *q*-packing (subspace codes)

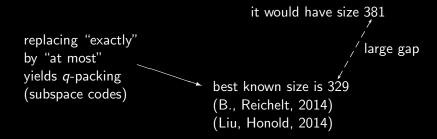


it would have size 381

replacing "exactly"
by "at most"
yields q-packing
(subspace codes)

best known size is 329 (B., Reichelt, 2014) (Liu, Honold, 2014)





what if $S_2(2,3,7)$ does exist?		

 $\mbox{what is the automorphism group?} \\ \mbox{what if $S_2(2,3,7)$ does exist?}$

what is the automorphism group?

what if $S_2(2,3,7)$ does exist?

exclude groups step by step

what is the automorphism group?

$$\mathop{\mathsf{A}}_ullet$$
 = $\mathop{\mathsf{Aut}}(\mathit{L}(\mathit{V}))$

$$\{1\}$$

consider only one subgroup of each conjugacy class

exclude groups step by step

1) choose representative *G* of some conjugacy class of *A*

what is the automorphism group?

A = Aut(L(V))

G ____

 $\{1\}$

consider only one subgroup of each conjugacy class

exclude groups step by step

1) choose representative
$$G$$
 of some conjugacy class of A

2) solve
$$\zeta_{t,k}^{G} x = [1, ..., 1]^{t}$$

exclude groups step by step

what is the automorphism group?

$$A = \operatorname{Aut}(L(V))$$

$$\{1\}$$

consider only one subgroup of each conjugacy class

2) solve
$$\zeta_{t,k}^{G} x = [1, ..., 1]^{t}$$

exclude groups step by step

what is the automorphism group?

$$\underset{\bullet}{A}=\operatorname{Aut}(L(V))$$

$$\{1\}$$

consider only one subgroup of each conjugacy class

1) choose representative G of some conjugacy class of A

2) solve
$$\zeta_{t,k}^{G} x = [1, ..., 1]^{t}$$

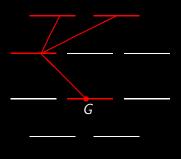
3) no solution

 \Rightarrow subgroups S with $G \leq S \leq A$ and their conjugacy classes cannot occur as groups of automorphisms

exclude groups step by step

what is the automorphism group?

$$A = \operatorname{Aut}(L(V))$$



 $\{1\}$

consider only one subgroup of each conjugacy class

systematic elimination of
subgroups (p-groups first)
and conjugacy classes yields

systematic elimination of subgroups (*p*-groups first) and conjugacy classes yields...

Theorem (B., Kiermaier, Nakić, 2014)

the automorphism group of $S_2(2,3,7)$ is either trivial or

generated by one of the following matrices						
ГО 1 О О О О О О	го 1 0 0 0 0 0 7	го 1 0 0 0 0 0 7	Γ1 1 0 0 0 0 0 ₇			
1000000	1100000	1100000	0110000			
0001000	0001000	0001000	0010000			
0010000	0011000	0011000	0001100			
0000010	0000010	0000100	0000110			
0000100	0000110	0000010	0000011			
[0000001]	[0000001]	[0000001]	[0000001]			
order 2	order 3	order 3	order 4			

we consider collections of disjoint designs		

we consider collections

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Theorem (B., Kohnert, Östergård, Wassermann, 2014) large sets $LS_a[N](t, k, n)$ for N > 2 do exist

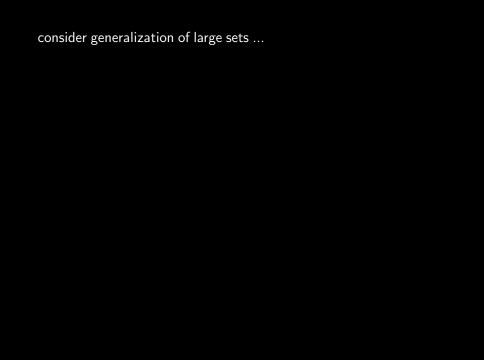
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$$LS_2[3](2,3,8)$$



$$\mathcal{S} \subseteq {V \brack k}$$
 is (N, t) -partitionable iff there is a partition $\mathcal{B}_1, \dots, \mathcal{B}_N$ of \mathcal{S} such that $\forall T \in {V \brack t}$ and $\forall i, j \mid \{B \in \mathcal{B}_i \mid T \subseteq B\} \mid = |\{B \in \mathcal{B}_j \mid T \subseteq B\}|$

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large set
$$LS_q[N](t, k, n)$$

for $S = \begin{bmatrix} V \\ L \end{bmatrix}$

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number can be different for different T

idea for recursive construction of large sets

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idea for recursive construction of large sets

– union and "joins" of (N,*)-partitionable sets are also (N,*)-partitionable

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 is $(N, *)$ -partitionable \Rightarrow large set

large set
$$LS_q[N](t, k, n)$$

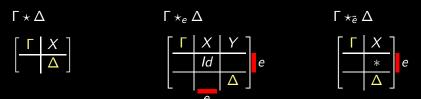
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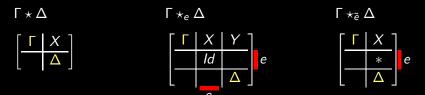
dentify subspaces with	
canonical generator matrices	
columns form base)	

$$\Gamma \star \Delta$$

$$\begin{array}{|c|c|c|c|}\hline \Gamma & X \\ \hline & \Delta \\ \hline \end{array}$$

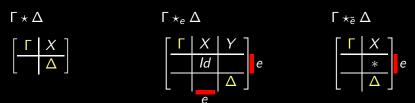


X, Y contain 0 at all pivot row positions of Γ , Id identity matrix



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$$\mathcal{S} \star \mathcal{T} := \{ \Gamma \star \Delta \mid \Gamma \in \mathcal{S}, \Delta \in \mathcal{T} \}$$

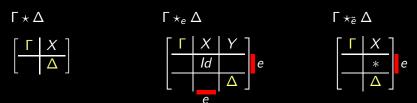


X, Y contain 0 at all pivot row positions of Γ , Id identity matrix

Lemma (B., Kiermaier, Kohnert, Laue, 2014)

1)
$$S, T \subseteq {V \brack k}$$
 (N, t)-partitionable, disjoint $S \cup T$ (N, t)-partitionable

$$\mathcal{S} \star \mathcal{T} := \{ \Gamma \star \Delta \mid \Gamma \in \mathcal{S}, \Delta \in \mathcal{T} \}$$

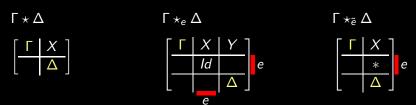


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Lemma (B., Kiermaier, Kohnert, Laue, 2014)

- 1) $\mathcal{S}, \mathcal{T} \subseteq {V \brack k}$ (N, t)-partitionable, disjoint
- $\Rightarrow S \cup \widetilde{T}(N, t)$ -partitionable
- 2) $S \subseteq \begin{bmatrix} V \\ k \end{bmatrix}$ (N, t)-partitionable, $\mathcal{T} \subseteq \begin{bmatrix} W \\ s \end{bmatrix}$ (N, r)-partitionable $\Rightarrow S \star \mathcal{T}$ (N, t + r + 1)-partitionable

$$\mathcal{S}\star\mathcal{T}:=\{\Gamma\star\Delta\mid\Gamma\in\mathcal{S},\Delta\in\mathcal{T}\}$$



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Lemma (B., Kiermaier, Kohnert, Laue, 2014)

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also valid for \star_e and $\star_{\tilde{e}}$

$$\mathcal{S} \star \mathcal{T} := \{ \Gamma \star \Delta \mid \Gamma \in \mathcal{S}, \Delta \in \mathcal{T} \}$$

there exist decompositions of ${V \brack k}$ into unions of joins corresponding to

generalized q-Vandermonde identities

$$\begin{bmatrix} \mathbb{F}_q^{10} \\ 3 \end{bmatrix} = \begin{bmatrix} \mathbb{F}_q^3 \\ 0 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^6 \\ 3 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^4 \\ 1 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^5 \\ 2 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^5 \\ 2 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^4 \\ 1 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^6 \\ 3 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^3 \\ 0 \end{bmatrix}$$

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$$\downarrow_{\mathsf{LS}_q[N](2,3,6)}$$

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$$\downarrow LS_q[N](2,3,6)$$

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$$\downarrow \qquad \qquad \downarrow$$

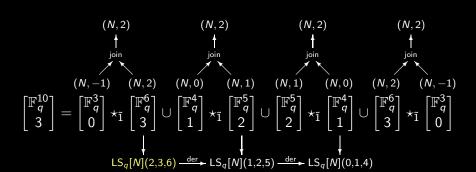
$$LS_q[N](2,3,6) \xrightarrow{\text{der}} LS_q[N](1,2,5)$$

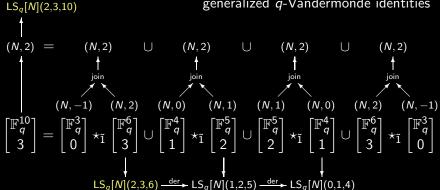
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$$\downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \qquad \qquad \downarrow \qquad \qquad \qquad \qquad$$

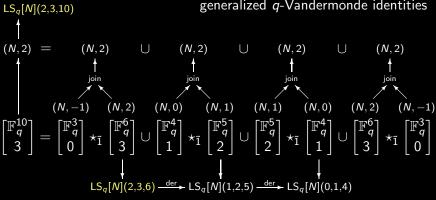
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there exist decompositions of ${V \brack k}$ into unions of joins corresponding to generalized q-Vandermonde identities



Theorem (B., Kiermaier, Kohnert, Laue, 2014)

the existence of $LS_q[N](2,3,6)$ implies the existence of $LS_q[N](2,k,n)$ for $n \ge 10$, $n \equiv 2 \mod 4$, $n-3 \ge k \ge 3$, $k \equiv 3 \mod 4$

there exist decompositions of ${V \brack k}$ into unions of joins corresponding to generalized q-Vandermonde identities

$$\mathsf{LS}_q[N](2,3,10) \qquad \qquad \mathsf{generalized} \ q\text{-Vandermonde identities}$$

$$(N,2) = (N,2) \cup (N,$$

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finally... the world of combinatorial q-analogs

finally... the world of combinatorial q-analogs

