A survey on designs over finite fields

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ALCOMA15

dedicated to our friend Axel Kohnert (1962–2013)

goals of this talk ...

goals of this talk ...

recall results
of the past 30 years

goals of this talk ...

recall results
of the past 30 years

focus on
constructions

goals of this talk ...

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computer results
group actions

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focus on
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provide list of
known construtions

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provide list of
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computer results
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describe some
new results

first, let's start with a typical slide on q-analogs ...

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V set of cardinality n

$P(V)$ lattice of subsets of V

$\binom{V}{k}$ set of k-subsets of V

$\binom{n}{k}$ binomial coefficient

$\mathrm{Aut}(P(V)) \simeq \mathrm{Sym}(V)$

$\mathrm{Anti}(P(V)) = \bar{\cdot} \circ \mathrm{Aut}(P(V))$

first, let's start with a typical slide on q-analogs ...

V set of cardinality n	V n-dimensional vector space over $\mathbb{F}_q$
$P(V)$ lattice of subsets of V	$L(V)$ lattice of subspaces of V
$\binom{V}{k}$ set of k-subsets of V	$\begin{bmatrix} V \\ k \end{bmatrix}$ set of k-subspaces of V
$\binom{n}{k}$ binomial coefficient	$\begin{bmatrix} n \\ k \end{bmatrix}_q$ q-binomial coefficent
$\mathrm{Aut}(P(V)) \simeq \mathrm{Sym}(V)$	$\mathrm{Aut}(L(V)) \simeq \mathrm{P\Gamma L}(V)$
$\mathrm{Anti}(P(V)) = \bar{\cdot} \circ \mathrm{Aut}(P(V))$	$\mathrm{Anti}(L(V)) = \cdot^{\perp} \circ \mathrm{Aut}(L(V))$

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take any incidence structure on sets,
replace sets by vector spaces,
and cardinalities by dimensions

take a combinatorial design ...

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$(V, \mathcal{B})$ is a t-(n, k, λ) design
iff $V = \{1, \ldots, n\}$, $\mathcal{B} \subseteq \binom{V}{k}$ blocks
$\forall T \in \binom{V}{t} : |\{B \in \mathcal{B} \mid T \subseteq B\}| = \lambda$

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number of blocks $|\mathcal{B}| = \lambda \frac{\begin{bmatrix} n \\ t \end{bmatrix}_q}{\begin{bmatrix} k \\ t \end{bmatrix}_q}$

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number of blocks $|\mathcal{B}| = \lambda \frac{\begin{bmatrix} n \\ t \end{bmatrix}_q}{\begin{bmatrix} k \\ t \end{bmatrix}_q}$

trivial design $\mathcal{B} = \begin{bmatrix} V \\ k \end{bmatrix}$ with $\lambda = \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q$

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a first example ...

a first example ... 1-$(4, 2, 3; 2)$

a first example ... $1\text{-}(4, 2, 3; 2)$

$$\begin{bmatrix}1&1\\1&0\\1&0\\0&0\end{bmatrix}\quad\begin{bmatrix}1&1\\1&0\\0&0\\1&0\end{bmatrix}\quad\begin{bmatrix}1&1\\0&0\\1&0\\1&0\end{bmatrix}\quad\begin{bmatrix}1&0\\1&1\\1&0\\0&0\end{bmatrix}\quad\begin{bmatrix}1&0\\1&1\\0&0\\1&0\end{bmatrix}$$

$$\begin{bmatrix}0&0\\1&1\\1&0\\1&0\end{bmatrix}\quad\begin{bmatrix}1&0\\1&0\\1&1\\0&0\end{bmatrix}\quad\begin{bmatrix}1&0\\0&0\\1&1\\1&0\end{bmatrix}\quad\begin{bmatrix}0&0\\1&0\\1&1\\1&0\end{bmatrix}\quad\begin{bmatrix}1&0\\1&0\\0&0\\1&1\end{bmatrix}$$

$$\begin{bmatrix}1&0\\0&0\\1&0\\1&1\end{bmatrix}\quad\begin{bmatrix}0&0\\1&0\\1&0\\1&1\end{bmatrix}\quad\begin{bmatrix}1&1\\1&1\\0&1\\0&1\end{bmatrix}\quad\begin{bmatrix}1&1\\0&1\\1&1\\0&1\end{bmatrix}\quad\begin{bmatrix}0&1\\1&1\\1&1\\0&1\end{bmatrix}$$

a first example ... $1\text{-}(4,2,3;2)$

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verification is quite uncomfortable ...

... but the incidence matrix makes it easy

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	0001,0111	0001,1011	0001,1101	0010,0111	0010,1011	0010,1110	0100,0111	0100,1101	0100,1110	1000,1011	1000,1101	1000,1110	1111,0011	1111,0101	1111,0110

... but the incidence matrix makes it easy

	0001,0111	0001,1011	0001,1101	0010,0111	0010,1011	0010,1110	0100,0111	0100,1101	0100,1110	1000,1011	1000,1101	1000,1110	1111,0011	1111,0101	1111,0110
1000															
0100															
0010															
0001															
1100															
1010															
1001															
0110															
0101															
0011															
1110															
1101															
1011															
0111															
1111															

... but the incidence matrix makes it easy

	0001, 0111	0001, 1011	0001, 1101	0010, 0111	0010, 1011	0010, 1110	0100, 0111	0100, 1101	0100, 1110	1000, 1011	1000, 1101	1000, 1110	1111, 0011	1111, 0101	1111, 0110
1000	1	1	1												
0100				1	1	1									
0010							1	1	1						
0001										1	1	1			
1100							1			1			1		
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0110	1											1			1
0101		1							1					1	
0011			1			1							1		
1110	1			1			1								
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row sum is $\lambda = 3$ for all rows

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extend action of $A = \mathrm{Aut}(L(V))$ to subsets of $L(V)$ by

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Orbit-Stabilizer-Theorem

$A(\mathcal{B}) \rightarrowtail\!\!\!\rightarrow A/A_{\mathcal{B}}$

$A_{g\mathcal{B}} = gA_{\mathcal{B}}g^{-1}$

$G = A_{\mathcal{B}} = A_{g\mathcal{B}} \Rightarrow g \in \mathrm{N}_A(G)$

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$\Rightarrow$ the normalizer of automorphims groups gives hints for classification issues (several papers by Laue)

$\Rightarrow$ candidates for possible automorphism groups can be reduced to different conjugacy classes

our 1-$(4, 2, 3; 2)$ design admits
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$\Rightarrow$

$$\begin{bmatrix} 3 & 0 \\ 2 & 1 \\ 3 & 0 \\ 0 & 3 \end{bmatrix}$$

condensed
G-incidence matrix
used to represent
design admitting G

some subgroups of $\mathrm{GL}(n, q)$ ...

Singer cycle
$\mathsf{S}(n,q) = \langle \sigma \rangle \simeq \mathbb{F}_{q^n}^*$

some subgroups of $\mathrm{GL}(n,q)$ ...

Singer cycle
$S(n,q) = \langle \sigma \rangle \simeq \mathbb{F}_{q^n}^*$

Frobenius automorphism
$F(n,q) = \langle \phi \rangle \simeq \mathrm{Aut}(\mathbb{F}_{q^n}/\mathbb{F}_q)$

some subgroups of $GL(n,q)$ ...

normalizer of
a Singer cycle
$\mathsf{N}(n,q) = \langle \sigma, \phi \rangle$

$\rtimes$

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some subgroups of $\mathrm{GL}(n,q)$ ...

if ℓ divides n then $\langle \sigma, \phi^{\ell} \rangle \simeq \mathsf{N}(n/\ell, q^{\ell})$
if n prime then $\mathsf{N}(n, q)$ is maximal in $\mathsf{GL}(n, q)$
$\mathsf{N}(n, q)$ is self-normalizing in $\mathsf{GL}(n, q)$

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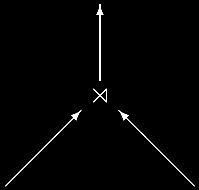

Singer cycle
$\mathsf{S}(n,q) = \langle \sigma \rangle \simeq \mathbb{F}^*_{q^n}$

Frobenius automorphism
$\mathsf{F}(n,q) = \langle \phi \rangle \simeq \mathrm{Aut}(\mathbb{F}_{q^n}/\mathbb{F}_q)$

Corollary
for primes n and q
different t-$(n,k,\lambda;q)$ designs
admitting $\mathsf{N}(n,q)$ as a group of
automorphisms are non-isomorphic

some subgroups of $\mathsf{GL}(n,q)$ ...

Thomas constructed the first non-trivial design
extended by Suzuki to an infinite series for all q

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$$S_r(T) := \langle b_0 \dots b_{r-1} \mid b_i \in T, 0 \le i < r \rangle$$
$$\mathcal{B}_r := \{S_r(T) \mid T \in \begin{bmatrix} V \\ 2 \end{bmatrix}\}$$

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Theorem (Suzuki, 1990)
for $n \geq 7$ with $(n, 4) = 1$ the set $\mathcal{B}_2$ defines a 2-$(n, 3, \begin{bmatrix} 3 \\ 2 \end{bmatrix}_q; q)$ design admitting $\mathrm{N}(n, q)$ as a group of automorphisms

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under which conditions does $\mathcal{B}_r$
define a 2-$(n, r+1, \begin{bmatrix} r+1 \\ 2 \end{bmatrix}_q; q)$ design?

Thomas constructed the first non-trivial design
extended by Suzuki to an infinite series for all q

$S_r(T) := \langle b_0 \dots b_{r-1} \mid b_i \in T, 0 \leq i < r \rangle$
$\mathcal{B}_r := \{ S_r(T) \mid T \in \begin{bmatrix} V \\ 2 \end{bmatrix} \}$

Theorem (Suzuki, 1990)
for $n \geq 7$ with $(n, 4) = 1$ the set $\mathcal{B}_2$ defines a 2-$(n, 3, \begin{bmatrix} 3 \\ 2 \end{bmatrix}_q; q)$ design admitting $\mathrm{N}(n, q)$ as a group of automorphisms

under which conditions does $\mathcal{B}_r$
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Theorem (Abe, Yoshiara, 1993)
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conjecture: $\mathcal{B}_3$ is the dual of $\mathcal{B}_2$ for $n = 7$ and all q

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$$
\begin{array}{cc}
 & K \in \begin{bmatrix} V \\ k \end{bmatrix} \\
\begin{bmatrix} V \\ t \end{bmatrix} \ni T & \begin{bmatrix} & \vdots & \\ \cdots & \zeta_{TK} & \\ & & \end{bmatrix}
\end{array}
\cdot \begin{bmatrix} \vdots \\ x_K \\ \vdots \end{bmatrix} = \begin{bmatrix} \lambda \\ \vdots \\ \lambda \end{bmatrix}
$$

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Theorem (Kramer, Mesner, 1976)
there exists a t-$(n, k, \lambda; q)$ design
admitting $G \leq \mathrm{Aut}(L(V))$
as a group of automorphisms
iff
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$$G(K) \quad \in G\backslash\!\backslash \begin{bmatrix} V \\ k \end{bmatrix}$$

$$G\backslash\!\backslash \begin{bmatrix} V \\ t \end{bmatrix} \ni G(T) \begin{bmatrix} & \vdots & \\ \cdots & \zeta^G_{TK} & \\ & & \end{bmatrix} \cdot \begin{bmatrix} \vdots \\ x_K \\ \vdots \end{bmatrix} = \begin{bmatrix} \lambda \\ \vdots \\ \lambda \end{bmatrix}$$

$$\zeta^G_{t,k} = (\zeta^G_{TK}) \qquad \text{with} \qquad \zeta^G_{TK} = \sum_{K' \in G(K)} \zeta(T, K')$$

we consider Suzuki's design...

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2-$(7, 3, 7; 2)$

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plain incidence matrix $\zeta_{2,3}$
2667×11811

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$$\zeta_{2,3}^{N(7,2)} = \begin{bmatrix} 5 & 3 & 1 & 3 & 2 & 3 & 1 & 2 & 2 & 3 & 2 & 1 & 1 & 2 & 0 \\ 1 & 2 & 0 & 1 & 3 & 2 & 5 & 3 & 3 & 3 & 2 & 1 & 2 & 2 & 1 \\ 1 & 2 & 0 & 3 & 2 & 2 & 1 & 2 & 2 & 1 & 3 & 5 & 4 & 3 & 0 \end{bmatrix}$$

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19 solutions (non-isomorphic)

a particular class of designs ...

t-$(n, k, \lambda; q)$ design $\mathcal{B}$ transitive iff $|A_{\mathcal{B}} \backslash\backslash \left[{V \atop 1}\right]| = 1$

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$\mathcal{B}$ non-trivial transitive t-$(n, k, \lambda; q)$ design, $t \geq 2$
– if $n = 6$ then $q \equiv 1 \bmod 3$ and $\mathrm{S}(6, q) \not\leq A_{\mathcal{B}} \leq \mathrm{N}(6, q)$
– if n prime then either $A_{\mathcal{B}} = \mathrm{S}(n, q)$ or $A_{\mathcal{B}} = \mathrm{N}(n, q)$

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Theorem (Miyakawa, Munemasa, Yoshiara, 1995)
[λ, number of non-isom. 2-$(7, 3, \lambda; q)$ designs $\mathcal{B}$ with $A_{\mathcal{B}} = N(n, q)$]

$q = 2$: $[3, 2]$, $[5, 14]$, $[7, 19]$, $[10, 30]$, $[12, 90]$, $[14, 55]$,
$[\lambda, 0]$ for $\lambda = 1, 2, 4, 6, 8, 9, 11, 13, 15$
$q = 3$: $[5, 22]$, $[\lambda, 0]$ for $\lambda = 1, 2, 3, 4$

further computer results for the binary field ...

further computer results for the binary field ...

Theorem (B., Kerber, Laue, S. Braun, 2005–2015)

t-$(n,k,\lambda;q)$	G	$\lvert\zeta_{t,k}^{G}\rvert$	λ
3-$(8,4,\lambda;2)$	$\mathrm{N}(8,2)$	53×109	11
	$\mathrm{N}(4,2^2)$	105×217	11, 15
2-$(11,3,\lambda;2)$	$\mathrm{N}(11,2)$	31×2263	245, 252
2-$(10,3,\lambda;2)$	$\mathrm{N}(10,2)$	20×633	15, 30, 45, 60, 75, 90, 105, 120
2-$(9,4,\lambda;2)$	$\mathrm{N}(9,2)$	11×725	21, 63, 84, 126, 147, 189, 210, 252, 273, 315, 336, 378, 399, 441, 462, 504, 525, 567, 588, 630, 651, 693, 714, 756, 777, 819, 840, 882, 903, 945, 966, 1008, 1029, 1071, 1092, 1134, 1155, 1197, 1218, 1260, 1281, 1323
2-$(9,3,\lambda;2)$	$\mathrm{N}(9,2)$	11×177	63
	$\mathrm{N}(3,2^3)$	31×529	21, 22, 42, 43, 63
	$\mathrm{N}(8,2)\times 1$	28×408	7, 12, 19, 24, 31, 36, 43, 48, 55, 60
	$\mathrm{M}(3,2^3)$	40×460	49
2-$(8,4,\lambda;2)$	$\mathrm{N}(4,2^2)$	15×217	21, 35, 56, 70, 91, 105, 126, 140, 161, 175, 196, 210, 231, 245, 266, 280, 301, 315
	$\mathrm{N}(7,2)\times 1$	13×231	7, 14, 49, 56, 63, 98, 105, 112, 147, 154, 161, 196, 203, 210, 245, 252, 259, 294, 301, 308
2-$(8,3,\lambda;2)$	$\mathrm{N}(4,2^2)$	15×105	21
2-$(7,3,\lambda;2)$	$\mathrm{N}(7,2)$	3×15	3, 5, 7, 10, 12, 14,
	$\mathrm{S}(7,2)$	21×93	3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
2-$(6,3,\lambda;2)$	$\langle\sigma^7\rangle$	77×155	3, 6

... also for $q = 3$, 4 and 5

t-$(n, k, \lambda; q)$	G	$\lvert\zeta_{t,k}^{G}\rvert$	λ
2-$(8, 3, \lambda; 3)$	N$(8, 3)$	41×977	52, 104, 156
2-$(7, 3, \lambda; 3)$	N$(7, 3)$	13×121	5, . . . , 60
2-$(6, 3, \lambda; 3)$	$\langle\sigma^2, \phi^2\rangle$	25×76	20
	$\langle\sigma^{17}, \phi\rangle$	93×234	16
	S$(5, 3)\times 1$	51×150	$\underline{8}$, 16, 20
	$\langle\sigma^2\rangle \times 1$	91×280	8, $\underline{12}$, 16, 20
2-$(6, 3, \lambda; 4)$	$\langle\sigma^3, \phi\rangle$	51×161	15, 35
	$\langle\sigma^3, \phi\rangle \times 1$	57×229	$\underline{10}$, $\underline{25}$, $\underline{30}$, 35
2-$(6, 3, \lambda; 5)$	N$(6, 5)$	53×248	78

further constructions ....

further constructions ....

$$t\text{-}(n, k, \lambda; q)$$

further constructions ....

$$t\text{-}\left(n, k, \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q - \lambda; q\right)$$

supplemented

$$t\text{-}(n, k, \lambda; q)$$

further constructions ....

$t\text{-}(n, k, \left[{n-t \atop k-t}\right]_q - \lambda; q)$

supplemented

$t\text{-}(n, k, \lambda; q)$

derived

$(t-1)\text{-}(n-1, k-1, \lambda; q)$

further constructions ....

Theorem (Suzuki, 1989)
$(V, \mathcal{B})$ t-$(n, k, \lambda; q)$ design
$i + j \leq t$, $P \in \begin{bmatrix} V \\ i \end{bmatrix}$, $H \in \begin{bmatrix} V \\ n-j \end{bmatrix}$

$$\Rightarrow |\{B \in \mathcal{B} \mid P \leq B \leq H\}| = \lambda \frac{\begin{bmatrix} n-j-i \\ k-i \end{bmatrix}_q}{\begin{bmatrix} n-t \\ k-t \end{bmatrix}_q}$$

t-$\left(n, k, \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q - \lambda; q\right)$

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t-$(n, k, \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q - \lambda; q)$

supplemented

t-$(n, k, \lambda; q)$

$(i = t-1, j = 0)$ reduced

derived

$(t-1)$-$(n, k, \lambda \frac{q^{n-t+1}-1}{q^{k-t+1}-1}; q)$

$(t-1)$-$(n-1, k-1, \lambda; q)$

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t-$(n,k,\begin{bmatrix} n-t \\ k-t \end{bmatrix}_q - \lambda;q)$

supplemented

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$(i=t-1, j=0)$ reduced

derived

$(t-1)$-$(n,k,\lambda\frac{q^{n-t+1}-1}{q^{k-t+1}-1};q)$

$(t-1)$-$(n-1,k-1,\lambda;q)$

dual
$(i=0, j=t, \cdot^{\perp})$

t-$(n,n-k,\lambda\begin{bmatrix} n-t \\ k \end{bmatrix}_q / \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q;q)$

further constructions ....

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$$\Rightarrow |\{B \in \mathcal{B} \mid P \leq B \leq H\}| = \lambda \frac{\begin{bmatrix} n-j-i \\ k-i \end{bmatrix}_q}{\begin{bmatrix} n-t \\ k-t \end{bmatrix}_q}$$

$$t\text{-}(n, k, \lambda; q)$$

supplemented:
$$t\text{-}\left(n, k, \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q - \lambda; q\right)$$

$(i = t-1, j = 0)$ reduced:
$$(t-1)\text{-}\left(n, k, \lambda \frac{q^{n-t+1}-1}{q^{k-t+1}-1}; q\right)$$

derived:
$$(t-1)\text{-}(n-1, k-1, \lambda; q)$$

dual $(i = 0, j = t, \cdot^{\perp})$:
$$t\text{-}\left(n, n-k, \lambda \begin{bmatrix} n-t \\ k \end{bmatrix}_q / \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q; q\right)$$

residual $(i = t-1, j = 1)$:
$$(t-1)\text{-}\left(n-1, k, \lambda \frac{q^{n-k}-1}{q^{k-t+1}-1}; q\right)$$

Theorem (Kiermaier, Laue, 2014)
the existence of designs for derived and residual parameters implies the existence of a design for the reduced parameters

$2\text{-}(9, 3, 21; 2)$

↑

by computer

↓

$2\text{-}(9, 4, 441; 2)$

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$2\text{-}(9, 4, 441; 2)$

der

res

$3\text{-}(10, 4, 21; 2)$

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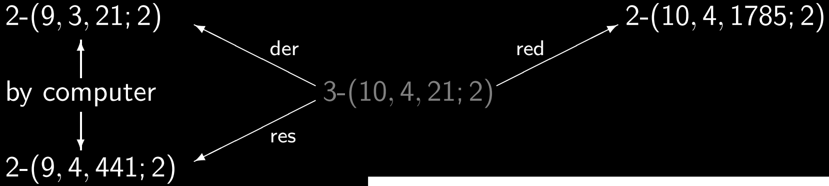
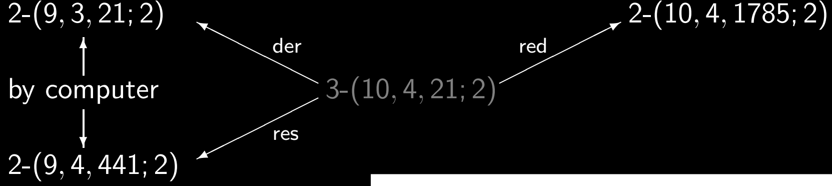


Theorem (Kiermaier, Laue, 2014)
the existence of designs for derived and residual parameters implies the existence of a design for the reduced parameters

Corollary (Kiermaier, Laue, 2014)
There exist designs with parameters
2-$(8, 4, \lambda; 2)$ for $\lambda = 63, 84, 147, 168, 189, 252, 273, 294$
2-$(10, 4, \lambda; 2)$ for $\lambda = 1785, 1870, 3570, 3655, 5355$
2-$(8, 4, 91\lambda; 3)$ for $\lambda = 5, 6, 7, \ldots, 60$



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further consequences?

take Suzuki's design

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$2\text{-}(7, 3, \begin{bmatrix}3\\2\end{bmatrix}_q; q)$

further consequences?

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$2\text{-}(7, 3, \begin{bmatrix}3\\2\end{bmatrix}_q; q)$

$\big\downarrow$ dual

$2\text{-}(7, 4, \begin{bmatrix}4\\2\end{bmatrix}_q; q)$

further consequences?

take Suzuki's design

$2\text{-}(7, 3, \begin{bmatrix}3\\2\end{bmatrix}_q; q)$

dual

der

$3\text{-}(8, 4, \begin{bmatrix}3\\2\end{bmatrix}_q; q)$

res

$2\text{-}(7, 4, \begin{bmatrix}4\\2\end{bmatrix}_q; q)$

further consequences?

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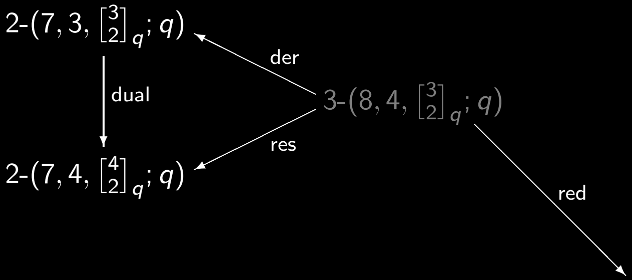
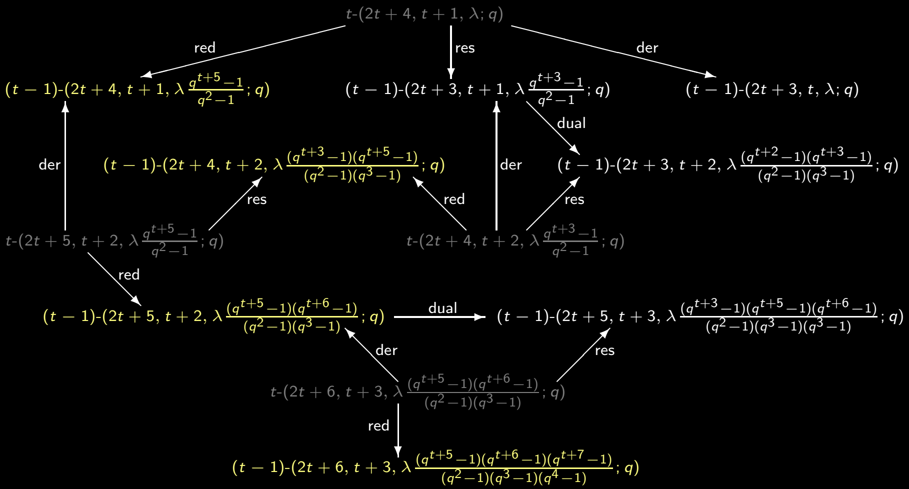
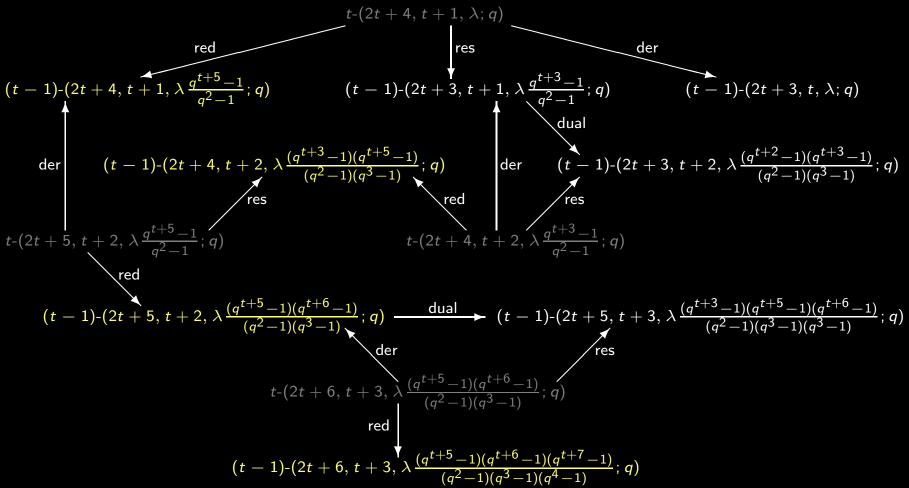


further consequences?

Corollary
there exists a family of
$2\text{-}(8, 4, \frac{(q^6-1)(q^3-1)}{(q^2-1)(q-1)}; q)$
designs for all prime powers q

$(t-1)$-$(2t+3,\, t+1,\, \lambda \frac{q^{t+3}-1}{q^2-1}; q)$ $(t-1)$-$(2t+3,\, t,\, \lambda; q)$


t-$(2t+4,\, t+1,\, \lambda;\, q)$
red
res
der
$(t-1)$-$(2t+4,\, t+1,\, \lambda\frac{q^{t+5}-1}{q^2-1};\, q)$
$(t-1)$-$(2t+3,\, t+1,\, \lambda\frac{q^{t+3}-1}{q^2-1};\, q)$
$(t-1)$-$(2t+3,\, t,\, \lambda;\, q)$
dual
der
$(t-1)$-$(2t+4,\, t+2,\, \lambda\frac{(q^{t+3}-1)(q^{t+5}-1)}{(q^2-1)(q^3-1)};\, q)$
der
$(t-1)$-$(2t+3,\, t+2,\, \lambda\frac{(q^{t+2}-1)(q^{t+3}-1)}{(q^2-1)(q^3-1)};\, q)$
res
red
res
t-$(2t+5,\, t+2,\, \lambda\frac{q^{t+5}-1}{q^2-1};\, q)$
t-$(2t+4,\, t+2,\, \lambda\frac{q^{t+3}-1}{q^2-1};\, q)$
red
$(t-1)$-$(2t+5,\, t+2,\, \lambda\frac{(q^{t+5}-1)(q^{t+6}-1)}{(q^2-1)(q^3-1)};\, q)$
dual
$(t-1)$-$(2t+5,\, t+3,\, \lambda\frac{(q^{t+3}-1)(q^{t+5}-1)(q^{t+6}-1)}{(q^2-1)(q^3-1)(q^3-1)};\, q)$
der
res
t-$(2t+6,\, t+3,\, \lambda\frac{(q^{t+5}-1)(q^{t+6}-1)}{(q^2-1)(q^3-1)};\, q)$
red
$(t-1)$-$(2t+6,\, t+3,\, \lambda\frac{(q^{t+5}-1)(q^{t+6}-1)(q^{t+7}-1)}{(q^2-1)(q^3-1)(q^4-1)};\, q)$



Corollary

There exist designs with parameters
$2\text{-}(10, 4, 85\lambda; 2)$, $2\text{-}(10, 5, 765\lambda; 2)$,
$2\text{-}(11, 5, 6205\lambda; 2)$, $2\text{-}(12, 6, 423181\lambda; 2)$
for $\lambda = 7, 12, 19, 21, 22, 24, 31, 36, 42, 43, 48, 49, 55, 60, 63$

computer results for $t = 3$ and $q = 2$

Itoh's infinite series of designs ...

Itoh's infinite series of designs ...

Theorem (Itoh, 1998)

there is a family of 2-$(n\ell, 3, q^3\frac{q^{n-5}-1}{q-1}; q)$ designs admitting $\mathrm{SL}(\ell, q^n)$ as group of automorphisms for $q \geq 2$, $\ell \geq 3$, $n \equiv 5 \bmod 6(q-1)$

Itoh's infinite series of designs ...

$$\zeta_{2,3}^{\mathrm{SL}(\ell,q^n)} = \left[\begin{array}{c|ccc|c|c} \zeta_{2,3}^{\mathrm{S}(n,q)} & a & & & 0 & 0 \\ & & \ddots & & & \\ & & & a & & \\ \hline 0 & & X & & Y & Z \end{array}\right]$$

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2-$(n, 3, \lambda; q)$ designs admitting $\mathrm{S}(n, q)$
can be extended to
2-$(n\ell, 3, \lambda; q)$ designs admitting $\mathrm{SL}(\ell, q^n)$
if λ is appropriate

Theorem (Itoh, 1998)
there is a family of 2-$(n\ell, 3, q^3 \frac{q^{n-5}-1}{q-1}; q)$ designs admitting $\mathrm{SL}(\ell, q^n)$ as group of automorphisms for $q \geq 2$, $\ell \geq 3$, $n \equiv 5 \bmod 6(q-1)$

extension possible if

$$\lambda = q(q+1)(q^3-1)s + q(q^2-1)t$$

for some integer $s \geq 0$, $t \in \{0, 1\}$ if $3|n$ and $t = 0$ otherwise extension possible if

$$\lambda = q(q+1)(q^3-1)s + q(q^2-1)t$$

for some integer $s \geq 0$, $t \in \{0, 1\}$ if $3|n$ and $t = 0$ otherwise

for $\ell \geq 3$, $n \equiv 5 \bmod 6(q-1)$
Suzuki's supplemented design
has index value λ of required form

further results from
Itoh's construction?

$q = 2$: $\lambda = 42s + 6t$
$q = 3$: $\lambda = 312s + 24t$

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computer constructions
of designs admitting $\mathrm{S}(n, q)$

$2\text{-}(8, 3, 21; 2) \overset{\text{supp}}{\Rightarrow} 2\text{-}(8, 3, 42; 2)$

$2\text{-}(9, 3, 42; 2)$

$2\text{-}(9, 3, 43; 2) \overset{\text{supp}}{\Rightarrow} 2\text{-}(9, 3, 84; 2)$

$2\text{-}(10, 3, 45; 2) \overset{\text{supp}}{\Rightarrow} 2\text{-}(10, 3, 210; 2)$

$2\text{-}(13, 3, 42s; 2)$ for $1 \leq s \leq 25$

$2\text{-}(8, 3, 52; 3) \overset{\text{supp}}{\Rightarrow} 2\text{-}(8, 3, 312; 3)$

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further results from
Itoh's construction?

Corollary

there exist families of designs
admitting $\mathrm{SL}(\ell, q^n)$ for $\ell \geq 3$

$2\text{-}(8\ell,3,42;2)$

$2\text{-}(9\ell,3,42s;2)$ for $1 \leq s \leq 2$

$2\text{-}(10\ell,3,210;2)$

$2\text{-}(13\ell,3,42s;2)$ for $1 \leq s \leq 25$

$2\text{-}(8\ell,3,312;3)$

$S_q(t, k, n)$ q-Steiner system
$= t$-$(n, k, 1; q)$ design

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Theorem

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Theorem
$S_q(1, k, n)$ exists if and only if k divides n

trivial q-Steiner systems

existence of non-trivial
q-Steiner systems ($t > 1$)
was long-standing issue ...

Theorem (B., Etzion, Östergård, Vardy, Wassermann, 2012)
q-Steiner systems do exist

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$S_2(2, 3, 13)$

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$S_2(2, 3, 13)$

automorphism group $N(13, 2)$
of order $13 \cdot (2^{13} - 1) = 106483$

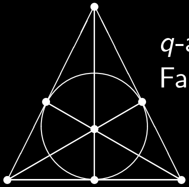
consisting of 15 orbits
of full length 106483

taken out of 25572
possible orbits

≥ 1050 disjoint solutions
all non-isomorphic

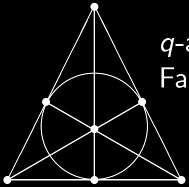
solved with Knuth's dancing links

$S_2(2, 3, 7)$?



q-analog of the Fano plane $S(2,3,7)$

$S_2(2,3,7)$?



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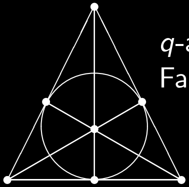
it would have size 381



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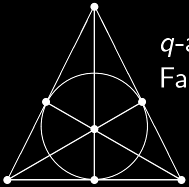
replacing "exactly"
by "at most"
yields *q*-packing
(subspace codes)



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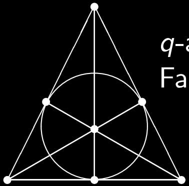
best known size is 329
(B., Reichelt, 2014)
(Liu, Honold, 2014)



q-analog of the Fano plane $S(2,3,7)$

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it would have size 381

large gap

replacing "exactly" by "at most" yields q-packing (subspace codes)

best known size is 329
(B., Reichelt, 2014)
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what if $S_2(2,3,7)$ does exist?

what is the automorphism group?

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exclude groups step by step

what is the automorphism group?

what if $S_2(2,3,7)$ does exist?

$A = \mathrm{Aut}(L(V))$

$\{1\}$

consider only one subgroup of each conjugacy class

exclude groups step by step

what if $S_2(2,3,7)$ does exist?

1) choose representative G of some conjugacy class of A

exclude groups step by step

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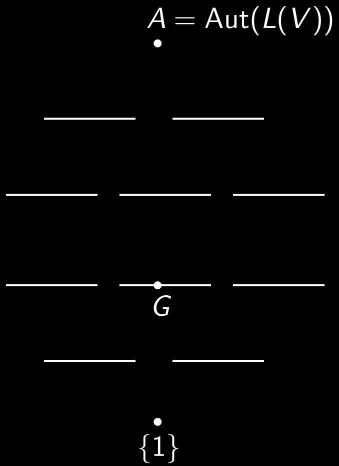

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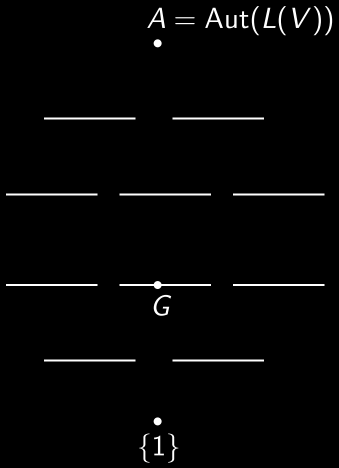

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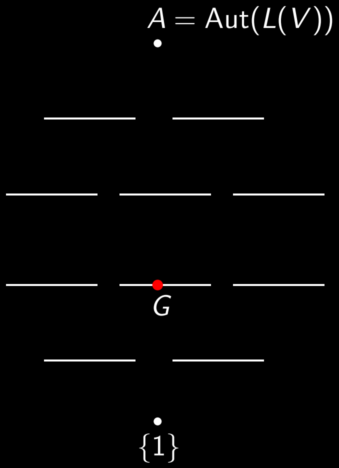

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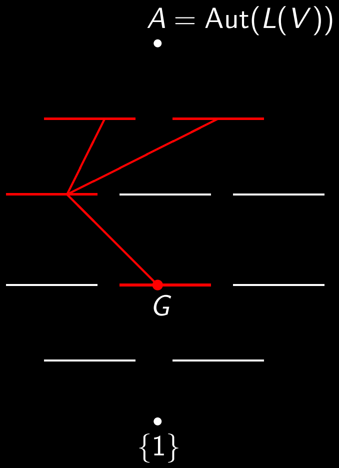
$\Rightarrow$ subgroups S with $G \leq S \leq A$ and their conjugacy classes cannot occur as groups of automorphisms

exclude groups step by step

what is the automorphism group?



consider only one subgroup of each conjugacy class

systematic elimination of
subgroups (p-groups first)
and conjugacy classes yields...

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subgroups (p-groups first)
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Theorem (B., Kiermaier, Nakić, 2014)
the automorphism group of $S_2(2,3,7)$ is either trivial or generated by one of the following matrices

$$\begin{bmatrix}0&1&0&0&0&0&0\\1&0&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&1&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&1&0&0\\0&0&0&0&0&0&1\end{bmatrix} \quad \begin{bmatrix}0&1&0&0&0&0&0\\1&1&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&1&1&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&1&1&0\\0&0&0&0&0&0&1\end{bmatrix} \quad \begin{bmatrix}0&1&0&0&0&0&0\\1&1&0&0&0&0&0\\0&0&0&1&0&0&0\\0&0&1&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{bmatrix} \quad \begin{bmatrix}1&1&0&0&0&0&0\\0&1&1&0&0&0&0\\0&0&1&0&0&0&0\\0&0&0&1&1&0&0\\0&0&0&0&1&1&0\\0&0&0&0&0&1&1\\0&0&0&0&0&0&1\end{bmatrix}$$

order 2   order 3   order 3   order 4

we consider collections
of disjoint designs ...

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an $\mathrm{LS}_q[N](t,k,n)$ large set is a partition of $\begin{bmatrix} V \\ k \end{bmatrix}$ into N disjoint t-$(n, k, \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q / N; q)$ designs

$N = 2$: halving

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$t\text{-}(n, k, \left[\begin{smallmatrix} n-t \\ k-t \end{smallmatrix}\right]_q / 2; q)$

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$$\Big\downarrow \text{supp}$$

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$$\begin{array}{ccc} t\text{-}(n, k, \left[{n-t \atop k-t}\right]_q/2; q) & \searrow & \\ \downarrow \text{supp} & & \mathrm{LS}_q[2](t, k, n) \\ t\text{-}(n, k, \left[{n-t \atop k-t}\right]_q/2; q) & \nearrow & \end{array}$$

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↓ supp

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examples

$\mathrm{LS}_3[2](2, 3, 6)$

$\mathrm{LS}_5[2](2, 3, 6)$

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Theorem (B., Kohnert, Östergård, Wassermann, 2014)
large sets $\mathrm{LS}_q[N](t, k, n)$ for $N > 2$ do exist

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$LS_5[2](2,3,6)$

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$\downarrow$ supp

$t\text{-}(n, k, \begin{bmatrix} n-t \\ k-t \end{bmatrix}_q /2; q)$

$\rightarrow LS_q[2](t,k,n)$

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$LS_2[3](2,3,8)$

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number can be different for different T

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$\Rightarrow$ $\begin{bmatrix} V \\ k \end{bmatrix}$ is $(N,*)$-partitionable $\Rightarrow$ large set

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identify subspaces with
canonical generator matrices
(columns form base)

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$\Gamma \star \Delta$

$$\left[\begin{array}{c|c} \Gamma & X \\ \hline & \Delta \end{array}\right]$$

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$\Gamma \star_e \Delta$

$$\left[\begin{array}{c|c|c} \Gamma & X & Y \\ \hline & Id & \\ \hline & & \Delta \end{array}\right] e$$

e

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X, Y contain 0 at all pivot row positions of Γ, Id identity matrix

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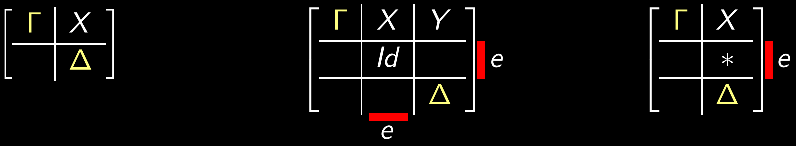

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Lemma (B., Kiermaier, Kohnert, Laue, 2014)

1) $\mathcal{S}, \mathcal{T} \subseteq \begin{bmatrix} V \\ k \end{bmatrix}$ (N, t)-partitionable, disjoint

$\Rightarrow \mathcal{S} \cup \mathcal{T}$ (N, t)-partitionable

$$\mathcal{S} \star \mathcal{T} := \{\Gamma \star \Delta \mid \Gamma \in \mathcal{S}, \Delta \in \mathcal{T}\}$$

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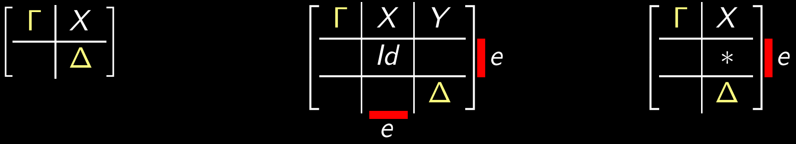

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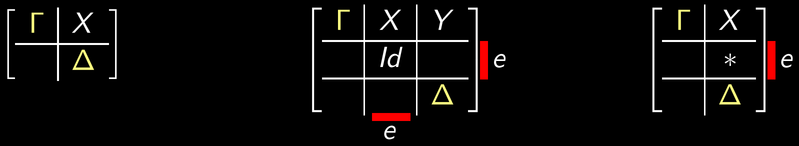
Lemma (B., Kiermaier, Kohnert, Laue, 2014)
1) $\mathcal{S}, \mathcal{T} \subseteq \begin{bmatrix} V \\ k \end{bmatrix}$ (N, t)-partitionable, disjoint
$\quad \Rightarrow \mathcal{S} \cup \mathcal{T}$ (N, t)-partitionable
2) $\mathcal{S} \subseteq \begin{bmatrix} V \\ k \end{bmatrix}$ (N, t)-partitionable, $\mathcal{T} \subseteq \begin{bmatrix} W \\ s \end{bmatrix}$ (N, r)-partitionable
$\quad \Rightarrow \mathcal{S} \star \mathcal{T}$ $(N, t + r + 1)$-partitionable

$$\mathcal{S} \star \mathcal{T} := \{\Gamma \star \Delta \mid \Gamma \in \mathcal{S}, \Delta \in \mathcal{T}\}$$

identify subspaces with
canonical generator matrices
(columns form base)

$\Gamma \star \Delta$ $\qquad$ $\Gamma \star_e \Delta$ $\qquad$ $\Gamma \star_{\tilde{e}} \Delta$



X, Y contain 0 at all pivot row positions of Γ, Id identity matrix

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also valid for $\star_e$ and $\star_{\tilde{e}}$

$$\mathcal{S} \star \mathcal{T} := \{\Gamma \star \Delta \mid \Gamma \in \mathcal{S}, \Delta \in \mathcal{T}\}$$

there exist decompositions of $\begin{bmatrix} V \\ k \end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities

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$$\begin{bmatrix} \mathbb{F}_q^{10} \\ 3 \end{bmatrix} = \begin{bmatrix} \mathbb{F}_q^{3} \\ 0 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{6} \\ 3 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{4} \\ 1 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{5} \\ 2 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{5} \\ 2 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{4} \\ 1 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{6} \\ 3 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{3} \\ 0 \end{bmatrix}$$

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$\downarrow$

$\mathrm{LS}_q[N](2,3,6)$

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$\mathrm{LS}_q[N](2,3,6) \xrightarrow{\text{der}} \mathrm{LS}_q[N](1,2,5)$

there exist decompositions of $\begin{bmatrix} V \\ k \end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities

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$$\mathrm{LS}_q[N](2,3,6) \xrightarrow{\text{der}} \mathrm{LS}_q[N](1,2,5) \xrightarrow{\text{der}} \mathrm{LS}_q[N](0,1,4)$$

there exist decompositions of $\begin{bmatrix} V \\ k \end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities

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$$\mathrm{LS}_q[N](2,3,6) \xrightarrow{\text{der}} \mathrm{LS}_q[N](1,2,5) \xrightarrow{\text{der}} \mathrm{LS}_q[N](0,1,4)$$

there exist decompositions of $\begin{bmatrix} V \\ k \end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities

$$\begin{matrix} & & (N,-1) & & (N,2) & & (N,0) & & (N,1) & & (N,1) & & (N,0) & & (N,2) & & (N,-1) \\ \begin{bmatrix} \mathbb{F}_q^{10} \\ 3 \end{bmatrix} & = & \begin{bmatrix} \mathbb{F}_q^{3} \\ 0 \end{bmatrix} & \star_{\tilde{1}} & \begin{bmatrix} \mathbb{F}_q^{6} \\ 3 \end{bmatrix} & \cup & \begin{bmatrix} \mathbb{F}_q^{4} \\ 1 \end{bmatrix} & \star_{\tilde{1}} & \begin{bmatrix} \mathbb{F}_q^{5} \\ 2 \end{bmatrix} & \cup & \begin{bmatrix} \mathbb{F}_q^{5} \\ 2 \end{bmatrix} & \star_{\tilde{1}} & \begin{bmatrix} \mathbb{F}_q^{4} \\ 1 \end{bmatrix} & \cup & \begin{bmatrix} \mathbb{F}_q^{6} \\ 3 \end{bmatrix} & \star_{\tilde{1}} & \begin{bmatrix} \mathbb{F}_q^{3} \\ 0 \end{bmatrix} \end{matrix}$$

$$\mathrm{LS}_q[N](2,3,6) \xrightarrow{\text{der}} \mathrm{LS}_q[N](1,2,5) \xrightarrow{\text{der}} \mathrm{LS}_q[N](0,1,4)$$

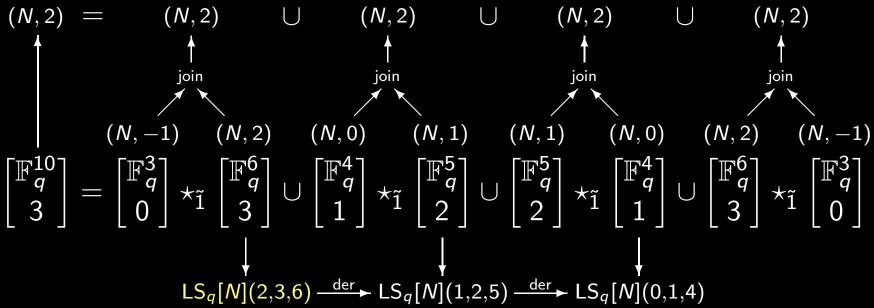
there exist decompositions of $\begin{bmatrix} V \\ k \end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities

$$(N,-1),\ (N,2) \xrightarrow{\text{join}} (N,2) \qquad (N,0),\ (N,1) \xrightarrow{\text{join}} (N,2) \qquad (N,1),\ (N,0) \xrightarrow{\text{join}} (N,2) \qquad (N,2),\ (N,-1) \xrightarrow{\text{join}} (N,2)$$

$$\begin{bmatrix} \mathbb{F}_q^{10} \\ 3 \end{bmatrix} = \begin{bmatrix} \mathbb{F}_q^{3} \\ 0 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{6} \\ 3 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{4} \\ 1 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{5} \\ 2 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{5} \\ 2 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{4} \\ 1 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{6} \\ 3 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{3} \\ 0 \end{bmatrix}$$

$$\mathrm{LS}_q[N](2,3,6) \xrightarrow{\text{der}} \mathrm{LS}_q[N](1,2,5) \xrightarrow{\text{der}} \mathrm{LS}_q[N](0,1,4)$$

there exist decompositions of $\begin{bmatrix} V \\ k \end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities



there exist decompositions of $\begin{bmatrix} V \\ k \end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities

$\mathrm{LS}_q[N](2,3,10)$

$$(N,2) = (N,2) \cup (N,2) \cup (N,2) \cup (N,2)$$

join: $(N,-1)$, $(N,2)$ | join: $(N,0)$, $(N,1)$ | join: $(N,1)$, $(N,0)$ | join: $(N,2)$, $(N,-1)$

$$\begin{bmatrix} \mathbb{F}_q^{10} \\ 3 \end{bmatrix} = \begin{bmatrix} \mathbb{F}_q^{3} \\ 0 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{6} \\ 3 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{4} \\ 1 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{5} \\ 2 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{5} \\ 2 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{4} \\ 1 \end{bmatrix} \cup \begin{bmatrix} \mathbb{F}_q^{6} \\ 3 \end{bmatrix} \star_{\tilde{1}} \begin{bmatrix} \mathbb{F}_q^{3} \\ 0 \end{bmatrix}$$

$$\mathrm{LS}_q[N](2,3,6) \xrightarrow{\text{der}} \mathrm{LS}_q[N](1,2,5) \xrightarrow{\text{der}} \mathrm{LS}_q[N](0,1,4)$$

there exist decompositions of $\begin{bmatrix}V\\k\end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities

$\mathrm{LS}_q[N](2,3,10)$

$$(N,2) = (N,2) \cup (N,2) \cup (N,2) \cup (N,2)$$

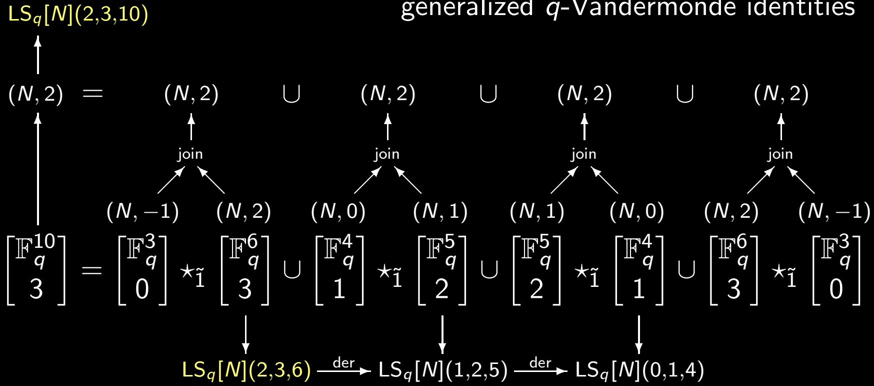
join: $(N,-1)$, $(N,2)$ | join: $(N,0)$, $(N,1)$ | join: $(N,1)$, $(N,0)$ | join: $(N,2)$, $(N,-1)$

$$\begin{bmatrix}\mathbb{F}_q^{10}\\3\end{bmatrix} = \begin{bmatrix}\mathbb{F}_q^{3}\\0\end{bmatrix} \star_{\tilde{1}} \begin{bmatrix}\mathbb{F}_q^{6}\\3\end{bmatrix} \cup \begin{bmatrix}\mathbb{F}_q^{4}\\1\end{bmatrix} \star_{\tilde{1}} \begin{bmatrix}\mathbb{F}_q^{5}\\2\end{bmatrix} \cup \begin{bmatrix}\mathbb{F}_q^{5}\\2\end{bmatrix} \star_{\tilde{1}} \begin{bmatrix}\mathbb{F}_q^{4}\\1\end{bmatrix} \cup \begin{bmatrix}\mathbb{F}_q^{6}\\3\end{bmatrix} \star_{\tilde{1}} \begin{bmatrix}\mathbb{F}_q^{3}\\0\end{bmatrix}$$

$$\mathrm{LS}_q[N](2,3,6) \xrightarrow{\text{der}} \mathrm{LS}_q[N](1,2,5) \xrightarrow{\text{der}} \mathrm{LS}_q[N](0,1,4)$$

Theorem (B., Kiermaier, Kohnert, Laue, 2014)
the existence of $\mathrm{LS}_q[N](2,3,6)$ implies the existence of $\mathrm{LS}_q[N](2,k,n)$ for $n \geq 10$, $n \equiv 2 \bmod 4$, $n-3 \geq k \geq 3$, $k \equiv 3 \bmod 4$

there exist decompositions of $\begin{bmatrix} V \\ k \end{bmatrix}$ into unions of joins corresponding to generalized q-Vandermonde identities



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$\Rightarrow$ infinite series for $N = 2$ and $q \in \{3,5\}$

finally... the world of combinatorial q-analogs

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