Towards a classification of special partial spreads and subspace codes

Alcoma 2015

Daniel Heinlein
Daniel.Heinlein@stmail.uni-bayreuth.de
University of Bayreuth

2015-03-19

joint work with T. Honold, M. Kiermaier, S. Kurz, A. Wassermann

contents

motivation

T. Honold, M. Kiermaier, S. Kurz classified all optimal solutions for a specific set of parameters in
Optimal binary subspace codes of length 6, constant dimension 3 and minimum distance 4

now we want to classify all optimal subspace codes of length 5, constant dimension 2 and minimum distance 4 using the field $\mathbb{F}_4$

direct approach is too hard, therefore objective:
find characteristics of optimal solutions

constant dimension codes

$G(n,q,k) := \{U \leq \mathbb{F}_q^n | \dim(U) = k\}$ is called Grassmannian

$C \subseteq G(n,q,k)$ is called constant dimension code with minimum distance

$$D(C) := \min\{2(k - \dim(U \cap V) | U \neq V \in C\} \geq 2d$$
$$\Leftrightarrow$$
$$\dim(U \cap V) \leq k - d \ \forall U \neq V \in C$$

use: $k = d = 2$, known as partial spread (in $PG(n-1,q)$)

objective: classification of all optimal solutions in $\mathbb{F}_4^5, k = d = 2$

intermediate objective: characteristics of optimal solution
$\Rightarrow$ spectrum

therefore: let C optimal constant dimension code be given

spectrum

y,a

$$y_H^C := \#\{V \in C | V \leq H\} \; \forall H \in G(n,q,n-1)$$
$$a_i^C := \#\{H \in G(n,q,n-1) | y_H^C = i\}$$

fact: $m \leq y_H^C \leq M \; \forall H \Rightarrow a_i^C = 0 \; \forall i \notin \{m, \ldots, M\}$

yields some constraints on a_i^C

spectrum

w,b

$$H^C := \{h \in G(n,q,1) | \forall V \in C : h \not\leq V\} \text{ is called holes}$$
$$w_H^C := \#\{h \in H^C | h \leq H\}\ \forall H \in G(n,q,n-1)$$
$$b_j^C := \#\{H \in G(n,q,n-1) | w_H^C = j\}$$

fact: $n = 5$ in addition to $k = d = 2$ yields:
$b_j^C = 0\ \forall q \not| j$
and $a_i^C = b_{q^2-q(i-1)}^C$

yields some constraints on b_j^C

spectrum

spectrum = sequence of a_i^C and b_j^C

characteristic of optimal solution C

from now on: $\mathbb{F}_4^5, k = d = 2$:
$(a_1^C, a_2^C, a_3^C, a_4^C, a_5^C, b_0^C, b_4^C, b_8^C, b_{12}^C, b_{16}^C)$
constraints $\Rightarrow$ spectrum depends only on two variables

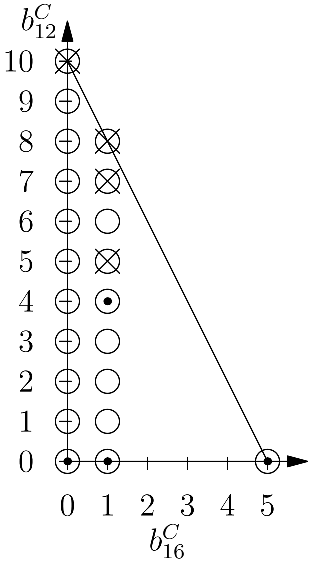
let C be *any* optimal solution

max z subject to

constraints for a_i^C and b_j^C

$a_i^C, b_j^C \in \mathbb{Z}$

with $z \in \{b_{16}^C, b_{12}^C, 2 \cdot b_{16}^C + b_{12}^C\}$
or
$(b_{16}^C, b_{12}^C) = \{0, \ldots, 10\}^2$ as
additional constraint



further approach

find more structure

compute representatives of orbit spaces

solve ILP using representatives as constraints

further approach

possibility for structure

fact:
every optimal solution contains structure:

$$\left\{ \{A, B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4\} \in \binom{G(5,4,2)}{9} \right|$$
$$\exists H_1 \neq H_2 \in G(5,4,4) : A \leq H_1 \cap H_2 \wedge$$
$$\left. B_i \leq H_1 \wedge C_j \leq H_2 \wedge \text{ only trivial intersections } \right\}$$

because $a_5 \geq 15$ and if this structure would be missing then any optimal solution would contain $\geq 15 \cdot 5 = 75 > 65$ elements

further approach

possibility for structure

fact:
every optimal solution that is missing in the diagram contains structure:

$$\left\{ \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}, h_{11}, h_{12}\} \in \binom{G(5,4,1)}{12} \middle| \exists H \in G(5,4,4) : h_i \leq H \right\}$$

because $b_{12} \geq 1$ for all missing entries

thank you for your attention

Honold, T., Kiermaier, M., & Kurz, S. (2013). Optimal binary subspace codes of length 6, constant dimension 3 and minimum distance 4. *arXiv preprint arXiv:1311.0464.*