Polar Grassmann Codes - Part 1

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joint work with L. Giuzzi

Alcoma15 – Kloster Banz 15-20 March 2015

Projective codes

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Ω: set of N points of $\mathrm{PG}(V)$, $V = V(K, q)$.

$$\updownarrow$$

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Theorem

Any semilinear collineation of $\mathrm{P\Gamma L}(K, q)$ stabilizing Ω induces automorphisms of $\mathcal{C}(\Omega)$.

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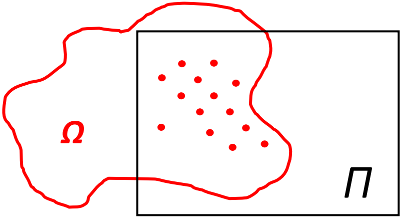
- $d_{min} = N - max_{\substack{\Pi \leq \mathrm{PG}(K-1,q) \\ codim\Pi = 1}} |\Pi \cap \Omega|$.



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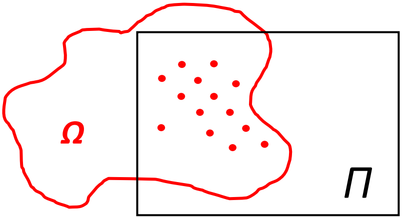

weights of $\mathcal{C}(\Omega) \leftrightarrow$ hyperplane sections of Ω.

Projective Grassmannians

$$V := V(m, q), \quad 1 \leq k < m$$

$\mathcal{G}_{m,k}$: k–Grassmannian of $\mathrm{PG}(V)$

* Points of $\mathcal{G}_{m,k}$: k–dimensional subspaces of V.
* Lines of $\mathcal{G}_{m,k}$: sets $l_{X,Y} := \{Z : X < Z < Y\}$ with $\dim(X) = k-1$, $\dim(Y) = k+1$.

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Grassmann or Plücker embedding of $\mathcal{G}_{m,k}$

$$e_k : \mathcal{G}_{m,k} \to \mathrm{PG}(\bigwedge^k V)$$
$$\langle v_1, \ldots, v_k \rangle \to \langle v_1 \wedge v_2 \wedge \cdots \wedge v_k \rangle$$

* $\dim(e_k) = \binom{m}{k}$

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Theorem [Nogin, 1996]

The parameters of $\mathcal{C}(\mathcal{G}_{m,k})$ are:

$$N = \begin{bmatrix} m \\ k \end{bmatrix}_q = \frac{(q^m-1)(q^m-q)\cdots(q^m-q^{k-1})}{(q^k-1)(q^k-q)\cdots(q^k-q^{k-1})},$$

$$K = \binom{m}{k}, \qquad d_{\min} = q^{(m-k)k}.$$

Minimum distance of Grassmann codes

k–multilinear alternating forms on V $\leftrightarrow$ Hyperplanes of $\bigwedge^k V$

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Remark

- *Minimum weight codewords in a Grassmann code correspond to k–multilinear alternating forms with a maximum number of totally isotropic spaces.*
- *When $k = 2$ these are non–null alternating forms with maximum radical.*

Polar Grassmannians

$\eta\colon V \times V \to \mathbb{F}_q$: non-singular bilinear form of Witt index n

$$\boxed{1 \leq k \leq n}$$

$\mathcal{P}_{n,k}$: k–polar Grassmannian associated to η

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$V := V(2n+1, q)$, η: non-singular symmetric form
$\Delta_{n,k} \subset \mathcal{G}_{2n+1,k}$: Orthogonal Grassmannian of rank n

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$\bar{\Delta}_{n,k}$: Symplectic Grassmannian of rank n

$$\bar{\varepsilon}_k : \begin{cases} \bar{\Delta}_{n,k} \to \mathrm{PG}(\overline{W}_k) \subseteq \mathrm{PG}(\bigwedge^k \overline{V}) \\ \langle v_1, \ldots, v_k \rangle \to \langle v_1 \wedge v_2 \wedge \cdots \wedge v_k \rangle \end{cases}$$

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Theorem [B. Cooperstein, JCTA 1998]

$\dim(\bar{\varepsilon}_k) = \binom{2n}{k} - \binom{2n}{k-2}$ for $1 \leq k \leq n$.

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Definition

- $\bar{\Delta}_{n,k}$: Symplectic Grassmannian
- $\mathcal{C}(\bar{\Delta}_{n,k}) := \mathcal{C}(\bar{\varepsilon}_k(\bar{\Delta}_{n,k}))$: Symplectic Grassmann codes.
  - ✓ $k = n \rightarrow$ Lagrangian Grassmann codes.

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* J. Carrillo-Pacheco, F. Zaldivar, *On Lagrangian Grassmann codes*, Design Codes Cryptography **60** (2011), 291-298.

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Symplectic Grassmann codes

For $1 \leq k \leq n$, the parameters of $\mathcal{C}(\bar{\Delta}_{n,k})$ are

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For $k = n > 2$, $d \leq q^{n(n+1)/2}$.
For $k = n = 2$, $d = q^3 - q$.

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Theorem[I.C., L. Giuzzi, ??]

If $k = n = 3$ then $d = q^6 - q^4$.
If $k = 2$ and $n \geq 2$ then $d = q^{4n-5} - q^{2n-3}$.

Orthogonal Grassmann codes

Theorem [I.C., Luca Giuzzi, 2013]

For $1 \leq k < n$, the parameters of $\mathcal{C}(\Delta_{n,k})$ are

$$N = \prod_{i=0}^{k-1} \frac{q^{2(n-i)} - 1}{q^{i+1} - 1}, \qquad K = \begin{cases} \binom{2n+1}{k} & \text{for } q \text{ odd} \\ \binom{2n+1}{k} - \binom{2n+1}{k-2} & \text{for } q \text{ even} \end{cases}$$

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Theorem

$MAut(\mathcal{C}(\Delta_{n,k})) \cong \Gamma O(2n+1, q)$.

Theorem [I.C., Luca Giuzzi, 2013]

The code $\mathcal{C}(\Delta_{2,2})$ arising from $\varepsilon_2(\Delta_{2,2})$ has parameters

$$N = (q^2+1)(q+1), \qquad K = \begin{cases} 10 & \text{for } q \text{ odd} \\ 9 & \text{for } q \text{ even} \end{cases}$$

$$d = q^2(q-1).$$

Theorem [I.C., Luca Giuzzi, 2013]

The code $\mathcal{C}(\Delta_{3,3})$ arising from $\varepsilon_3(\Delta_{3,3})$ has parameters

$$\left.\begin{array}{rl} N = (q^3+1)(q^2+1)(q+1), & K = 35, \\ d = q^2(q-1)(q^3-1) & \end{array}\right\} \quad \text{for } q \text{ odd}$$

and

$$\left.\begin{array}{rl} N = (q^3+1)(q^2+1)(q+1), & K = 28, \\ d = q^5(q-1) & \end{array}\right\} \quad \text{for } q \text{ even.}$$

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maximum number of lines being simultaneously totally singular for the quadratic form η (defining $\Delta_{n,2}$) and totally isotropic for a (degenerate) alternating form of V (defining a hyperplane of $\bigwedge^2 V$).

Theorem [I.C., Luca Giuzzi, A. Pasini, ??]

For q odd, the code $\mathcal{C}(\Delta_{n,2})$ is a $[N, K, d_{min}]$-code with

$$N = \frac{(q^{2n}-1)(q^{2n-2}-1)}{(q-1)(q^2-1)}, \quad K = \binom{2n+1}{2},$$

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- This applies also for $\Delta_{2,2}$.

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Encoding/Decoding $\rightarrow$ see Luca's talk